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Fixed Cost Allocation in the Presence of Undesirable Outputs in DEA

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Abstract

In this paper, we present a fixed cost allocation plan based on a Directional Distance Function (DDF) model in the presence of undesirable outputs in Data Envelopment Enalysis (DEA). The use of a DDF help for the accommodation of undesirable indicators in their original form. We propose a fixed cost allocation based on the the principle of full-efficient mechanism. This approach guarantees that the efficiency scores of Decision-Making Units (DMUs) will be equql to one after the allocation of fixed cost. By choosing different direction vectors, we can flexibly change the fixed cost allocation plan and the cost allocated to the units will also change. The proposed fixed cost allocation plan allocated cost among efficient and inefficient units. We illustrate the results of the proposed approach with a numerical example, and the results of the models are presented.

Keywords: Data envelopment analysis, Fixed cost allocation, Undesirable outputs.

1 | Introduction

DEA is a nonparametric approach with a structured mathematical programming framework. Using this technique, we assess the relative efficiency of homogeneous Decision-Making Units (DMUs) [1]. The higher the output and the lower the input of a DMU, the more favorable the performance assessment. However, in some situations, certain indicators considered as outputs may be achieved due to the way they are produced, even though their nature is such that they are undesirable, meaning the lower their values, the better. In real-world practice, there might be both desirable and undesirable outputs. For example, air pollution is an undesirable output of the production process in various companies. There are several approaches to dealing with undesirable outputs, some of which are mentioned here. The simplest approach is to ignore them in the

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evaluation process, but this may lead to misleading results [2]. Since smaller amounts of undesirable outputs are more desirable, some methods consider them as input indicators [3]. There are different approaches to treating them as outputs, and in this regard, some terminology has been proposed. The term "strong disposability of undesirable outputs" is used when they can be freely disposed of, while "weak disposability" is used when reducing their production depends on reducing the production of desirable outputs [4].

The first approach is based on the indirect method, initially proposed by Charnes et al. [1] and later modified by Seiford and Zhu [5]. This method incorporates undesirable outputs into efficiency assessment. In this method, Mis is assumed to be the maximum value of undesirable outputs, and the values of other undesirable outputs are evaluated by subtracting them from M. The second approach is based on the direct method, proposed by Chambers et al. [6] and later by Chung et al. [7]. Both approaches have significant limitations. The direct approach is limited by variations in the initial data, while the indirect approach faces issues such as exceeding the production possibility set and having a descending slope. Zanella et al. [8] proposed a new model derived from Shepherd et al. [9], which avoids the limitations of previous methods and evaluates efficiency in the presence of undesirable outputs using the Directional Distance Function (DDF).

For example, Maghbouli et al. [10] evaluated the efficiency of 39 Spanish airports while considering undesirable outputs. Dang [11] proposed a complete ranking of DMUs with undesirable outputs. Khalili Damghani and Shahmir [12] evaluated the efficiency of electricity generation and distribution companies using undesirable outputs. Khoshandam et al. [13] proposed a method to determine final substitution rates in Data Envelopment Analysis (DEA) with undesirable outputs. Pishgar-Komleh et al. [14] used six approaches to assess the winter wheat cropping system in Poland: 1) ignoring undesirable outputs, 2) slack-based measurement DEA with undesirable outputs, 3) treating undesirable outputs as inputs in the DEA model, 4) impact rate, 5) data transformation, and 6) ratio model. Chambers et al. [15], [6] studied the relationship between directional technology, distance function, and profit function, and applied the directional technology distance function in various economic settings. Fare et al. [16] used the directional output distance function to model the joint production of good and bad outputs and the reduced disposability of bad outputs imposed by regulations in the utility industry. Portela et al. [17] proposed the Range Directional Model (RDM) for handling positive and negative data in DEA. Sahoo et al. [18] extended value-based models in a directional DEA framework to develop new directional cost- and revenue-based measures of efficiency in the banking industry. Lee and Choi [19] used non-radial DDF to evaluate greenhouse gas performance by decomposing technical efficiency into pure technical efficiency and scale efficiency. Yang et al. [20] developed a DEA-based DDF model to investigate the appropriate (or best) direction for measuring efficiency. Pastor et al. [21] introduced a new Malmquist productivity index by modifying the conventional DDF.

The problem of allocating fixed costs often arises in real-world situations when multiple DMUs share a common platform. An example provided by Cook and Zhu [22] is the allocation of a manufacturer's advertising costs to local retailers. Another example is the allocation of a bank's joint television or newspaper advertising costs to its branches. The key challenge in allocating fixed costs is designing an optimal allocation plan to distribute the cost among multiple DMUs. So far, most DEA studies have focused on fixed cost allocation based on the efficiency conservation principle or the efficiency maximization principle. It should be noted that the efficiency of a DMU is defined here in relative terms, meaning it is measured in comparison to other DMUs. The efficiency conservation principle states that the efficiency of DMUs does not change before and after allocation. Cook and Kress [23] were the first to study fixed cost allocation using DEA. Their proposed method allocates costs by solving linear programming problems based on the efficiency conservation principle and the Pareto minimization principle. Lin [24] proved that the method proposed by Cook and Zhu [23] has no feasible solution under certain constraints. To achieve a feasible allocation scheme, Lin [24] improved Cook and Zhu's [23] method and set output targets based on the fixed cost amount allocated to each DMU. Additionally, Lin [24] proposed a DEA method for cost allocation and joint revenue distribution among DMUs, reflecting the relative efficiency and input-output scale of the DMUs. The efficiency maximization principle states that the efficiency of all DMUs will improve after cost allocation.

Beasley [25] presented the first cost allocation method based on this principle. Later, Si et al. [26] extended Beasley's [25] work. Li et al. [27] proposed that each DMU should propose an allocation plan to penalize itself, ensuring the acceptance of the allocation plan. Considering the game relations in the allocation process, Li [28] proposed a collaborative game-based approach for cost allocation. To ensure the uniqueness of the allocation result, Chu et al. [29] defined the concept of utility for each DMU and obtained the cost allocation result by maximizing the minimum utility. Lin and Chen [30] allocated fixed costs as a complement to other cost inputs based on the DEA method, and this method was extended to two-stage systems by Zhu et al. [31]. Zhang et al. [32] combined game theory and DEA to solve the problem of transfer cost allocation. Considering the competitive and cooperative relationships between DMUs, Xu et al. [33] presented a unique cost allocation plan based on DEA from the perspective of inequality aversion.

The main contribution of this paper is as follows: we present a fixed cost allocation plan in the presence of undesirable outputs. This plan based on the strategy of full-efficient mechanism. We considering fixed costs as a new input in the DDF DEA model. We propose an algorithm to determine the allocated cost to DMUs by considering all efficient and inefficient DMUs

The structure of this paper is orgonized as follows: in the second section, we introduce the DDF model in DEA to consider the undesirable outputs. In the third section, we present a fixed cost allocation plan in the presence of undesirable outputs based on the DDF model. In the fourth section, we illustrate the results of the proposed approach with a numerical example, we propose the results of our research in the last section.

2 | DDF Approach to Treat Undesirable Outputs

Let n DMUs as $\mathrm{DMU_j} = (X_j, U_j, Y_j)$. Each $\mathrm{DMU_j}$ consumes input vector $X_j = (x_{1j}, \dots, x_{mj}) \in R_+^m$ to produce $Y_j = (y_{1j}, \dots, y_{sj}) \in R_+^s$ as desirable output and $U_j = (u_{1j}, \dots, u_{lj}) \in R_+^l$ as undesirable output. Chung et al. [6] proposed *Model (1)* by considering weak disposability of undesirable outputs for measuring of $\mathrm{DMU_o}$ unit under evaluation as follows.

$$\begin{aligned} &\beta_o^* = \mathrm{Max}\ \beta_o,\\ &\text{s.t.}\ \ \textstyle\sum_{j=1}^{n} \mu_{jo}\, x_{ij} \le x_{io} - \beta g_i^x,\ i = 1, \dots, m,\\ &\qquad \textstyle\sum_{j=1}^{n} \mu_{jo}\, u_{kj} \le u_{ko} - \beta g_k^u,\ k = 1, \dots, l,\\ &\qquad \textstyle\sum_{j=1}^{n} \mu_{jo}\, y_{rj} \ge y_{ro} + \beta g_r^y,\ r = 1, \dots, s,\\ &\qquad\quad \mu_{jo} \ge 0,\ j = 1, \dots, n,\ \beta \text{ free sign.} \end{aligned} \tag{1}$$

In Model (1), x_{ij} are the inputs used by the $\mathrm{DMU_j}$, $j = 1, \dots n$ to produce y_{rj} desirable outputs and u_{kj} undesirable outputs. The vector $g = (g_i^x, g_k^u, g_r^y, i = 1, \dots, m,\ k = 1, \dots, l,\ r = 1, \dots, s)$ show direction of change of inputs, undesirable outputs and desirable outputs. The μ_{jo} is intensity variables. The factor β_o show the extent of the DMU's inefficiency. It corresponds to the maximal feasible expansion of desirable outputs and contraction of inputs and undesirable outputs that can be achieved simultaneously.

It is obvious that always $\beta_o^* \ge 0$, if $\beta_o^* > 0$ then $\mathrm{DMU_o}$ is efficient; otherwise, $\mathrm{DMU_o}$ is inefficient. The three commonly used predefined directions (input- oriented, output-oriented, and proportional) considered in this study. If we put $g = (X_j, 0,0)$, then we have *Model (1)* in the input oriented, and the efficiency of $\mathrm{DMU_o}$ is calculated as $1 - \beta_o^*$. If we put $g = (0, U_j, Y_j)$, then we have *Model (2)* in the output oriented, and the efficiency of $\mathrm{DMU_o}$ is calculated as $\frac{1}{1+\beta_o^*}$. If we put $g = (X_j, U_j, Y_j)$, then we have *Model (2)* in the mix oriented, and the efficiency of $\mathrm{DMU_o}$ is calculated as $1 - \beta_o^*$. It is obvious that always $0 < \beta_o^* < 1$.

By selecting three direction namely $g = (X_j, U_j, Y_j)$ as proportional direction, the dual of *Model (1)*, corresponding to the multiplier formulation,is shown in *Model (2)*.

$$\begin{aligned} \min \ & -\sum_{r=1}^{s} \gamma_r y_{ro} + \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io,} \\ \text{s.t.} \ & \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{l} w_k u_{ko} = 1, \\ & \sum_{k=1}^{l} w_k u_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{l} w_k u_{kj} \le 0, \quad j = 1, \dots, n, \\ & \gamma_r \ge 0, w_k \ge 0, v_i \ge 0, \ \ i = 1, \dots, m, \ k = 1, \dots, l, \ r = 1, \dots, s. \end{aligned} \tag{2}$$

Similar to the directional *Model (1)* the value of the objective function at the optimal solution to *Model (2)* corresponds to the maximal feasible improvement to the desirable and undesirable outputs that can be achieved simultaneously.

3 | Allocation of Fixed Costs with Undesirable Outputs based on the DDF Model

Let we want to distribute a total fixed cost R among units. Each DMU is allocated a non-negative cost R_j such that $\sum_{j=1}^{n} R_j = R, R_j \ge 0, j = 1, \dots, n$. The cost allocated to each DMU_j is considered as a new input. To proceed, we would like to introduce how to resolve the fixed cost allocation problem based on *Model (2)*.

To ultimately identify such an allocation, following Beasley [25] and Li et al. [34], we can treat the allocated cost R_j as an additional input to DMUj. As such, the fixed cost allocation of DMU_o can be determined by the *Model (3)*.

$$\begin{aligned} \min \ & -\sum_{r=1}^{s} \gamma_r y_{ro} + \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + v_{m+1} R_o, \\ \text{s.t.} \ & \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{l} w_k u_{ko} + v_{m+1} R_o = 1, \\ & \sum_{k=1}^{l} w_k u_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{l} w_k u_{kj} - v_{m+1} R_j \le 0, \quad j = 1, \dots, n, \\ & \sum_{j=1}^{n} R_j = R, R_j \ge 0, j = 1, \dots, n, \\ & \gamma_r \ge 0, w_k \ge 0, v_i \ge 0, \ \ i = 1, \dots, m, \ k = 1, \dots, l, \ r = 1, \dots, s. \end{aligned} \tag{3}$$

As can be seen in *Model (3)*, the problem is nonlinear because of the term $v_{m+1} R_j$. Let $v_{m+1} R_j = \pi_j, j = 1, \dots n$, *Model (3)* will be as follows:

$$\begin{aligned} \min \ & -\sum_{r=1}^{s} \gamma_r y_{ro} + \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \pi_{o,} \\ \text{s.t.} \ & \sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{l} w_k u_{ko} + \pi_o = 1, \\ & \sum_{k=1}^{l} w_k u_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{l} w_k u_{kj} - \pi_o \le 0, \quad j = 1, \dots, n, \\ & \sum_{j=1}^{n} \pi_j = v_{m+1} R, \pi_j \ge 0, j = 1, \dots, n, v_{m+1} \ge 0, \\ & \gamma_r \ge 0, w_k \ge 0, v_i \ge 0, \ \ i = 1, \dots, m, \ k = 1, \dots, l, \ r = 1, \dots, s. \end{aligned} \tag{4}$$

For each DMU_o, one can solve *Model (4)* to determine its corresponding the fixed cost allocation score separately. Suppose $\left(\gamma_r^*, w_k^*, v_i^*, v_{m+1}^*, \pi_j^*, r = 1, \dots, s, \ k = 1, \dots, l, i = 1, \dots m\right)$ is an optimal sulotion of *Model (4)*. Then the fixed cost allocation of DMU_o for all DMUs can be determined $R_j^* = \frac{\pi_j^*}{v_{m+1}^*}, j = 1, \dots . n$.

Theorem 1. There always exists a feasible $(\pi_1, \dots, \pi_n)$ that the fixed cost allocation score of each DMU_o determined by *Model (4)* is equal to unity, i.e., the the fixed cost allocation mechanism makes all DMUs efficient (also called full-efficient mechanism).

Proof: We can prove *Theorem 1* in an analogous way of Si et al. [26].

Various preference objectives can be included to realize a more practical the fixed cost allocation scheme.

We firstly normalize the input and output measures as follows:

$$\hat{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, \ \hat{u}_{kj} = \frac{u_{kj}}{\sum_{k=1}^{l} u_{kj}}, \ \hat{y}_{rj} = \frac{y_{rj}}{\sum_{r=1}^{s} y_{rj}}. \tag{5}$$

Let

$$\rho_j = \frac{(\sum_{i=1}^{m}\hat{x}_{ij})(\sum_{k=1}^{l}\hat{u}_{kj})(\sum_{r=1}^{s}\hat{y}_{rj})}{\sum_{j=1}^{n}\left((\sum_{i=1}^{m}\hat{x}_{ij})(\sum_{k=1}^{l}\hat{u}_{kj})(\sum_{r=1}^{s}\hat{y}_{rj})\right)}, \quad j = 1, \dots, n. \tag{6}$$

hat ρ_j can be interpreted as the proportion allocated to DMU_j. However, the cost proportions assigned to each DMU may not always be fully satisfied. We put $R_j^o = \rho_j R$. Then R_j^o is represented as the preferential cost allocated to DMUj such that $\sum_{j=1}^{n} R_j^o = R$. To this end, we determine the final allocation scheme by introducing a set of deviation variables to minimize the total deviations of all DMUs. The following generalized allocation model can be formulated such that preferential information of each individual DMU are considered.

$$\begin{aligned}
\min\ & \textstyle\sum_{j=1}^{n}\left|\pi_j - R_j^o\right|, \\
\text{s.t.}\ & \textstyle\sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{l} w_k u_{ko} + \pi_o = 1, \\
& \textstyle\sum_{k=1}^{l} w_k u_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{l} w_k u_{kj} - \pi_j \le 0, \quad j = 1, \dots, n, \\
& \textstyle\sum_{j=1}^{n} \pi_j = v_{m+1} R,\ \pi_j \ge 0, j = 1, \dots, n,\ v_{m+1} \ge 0, \\
& \gamma_r \ge 0, w_k \ge 0, v_i \ge 0, \quad i = 1, \dots, m,\ k = 1, \dots, l,\ r = 1, \dots, s.
\end{aligned} \tag{7}$$

We can linearization *Model (7)*, for this goal, we put $\left|\pi_j - R_j^o\right| + \pi_j - R_j^o = 2a_j$ and $\left|\pi_j - R_j^o\right| - \pi_j + R_j^o = 2b_j$, then $\sum_{j=1}^{n}(a_j + b_j) \le 2R$, then *Model (7)* can be easily reformulated as *Model (8)*.

$$\begin{aligned}
\min\ & \textstyle\sum_{j=1}^{n}(a_j + b_j), \\
\text{s.t.}\ & \textstyle\sum_{k=1}^{l} w_k u_{ko} + \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{l} w_k u_{ko} + \pi_o = 1, \\
& \textstyle\sum_{k=1}^{l} w_k u_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{l} w_k u_{kj} - \pi_j \le 0, \quad j = 1, \dots, n, \\
& \textstyle\sum_{j=1}^{n} \pi_j = v_{m+1} R,\ \pi_j \ge 0, j = 1, \dots, n,\ v_{m+1} \ge 0, \\
& \textstyle\sum_{j=1}^{n}(a_j + b_j) \le 2R, \\
& \pi_j - R_j^o = a_j + b_j, j = 1, \dots, n, \\
& \gamma_r \ge 0, w_k \ge 0, v_i \ge 0, \quad i = 1, \dots, m,\ k = 1, \dots, l,\ r = 1, \dots, s.
\end{aligned} \tag{8}$$

We obtain fixed cost allocation plane by solving *Model (8)*. To obtain the optimal solution from the previous models, we can inversely consider the change in the applied variables in the model.

4 | Numerical Illustration

To show the results of the models presented in this paper for the fixed cost allocation scheme, we use a numerical example from previous DEA studies. This dataset including 12 DMUs, each consuming three inputs to produce two desirable outputs and one undesirable output. We assume that we want to allocate a fixed cost of 100 units among the DMUs.

Table 1. Data set and the results of model (2).

DMUs	Input 1	Input 2	Input 3	Desirable Output 1	Desirable Output 2	Undesirable Output 1	The Efficiency Scores of Model (2)
1	350	39	9	67	751	5	1
2	298	26	8	73	611	25	0.9583
3	422	31	7	75	584	22	0.8953
4	281	16	9	70	665	10	1
5	301	16	6	75	445	30	1
6	360	29	17	83	1070	22	0.9973
7	540	18	10	72	457	20.5	0.9081
8	276	33	5	78	590	21.6	1
9	323	25	5	75	1074	25	1
10	444	64	6	74	1072	27	0.9248
11	323	25	5	25	350	23	0.5143
12	444	64	6	104	1199	25	1

As can be seen, some DMU1, DMU4, DMU5, DMU8, DMU9 and DMU12 are efficient and others DMUs are inefficient based on the *Model (2)* and three direction.

Table 2 show the scale of inputs and outputs by *Relation (5).*

Table 2. Scale of data.

DMUs	Input 1	Input 2	Input 3	Desirable Output 1	Desirable Output 2	Undesirable Output 1
1	0.0802	0.1010	0.0968	0.0769	0.0847	0.0195
2	0.0683	0.0674	0.0860	0.0838	0.0689	0.0976
3	0.0967	0.0803	0.0753	0.0861	0.0659	0.0859
4	0.0644	0.0415	0.0968	0.0804	0.0750	0.0390
5	0.0690	0.0415	0.0645	0.0861	0.0502	0.1171
6	0.0825	0.0751	0.1828	0.0953	0.1207	0.0859
7	0.1238	0.0466	0.1075	0.0827	0.0515	0.0800
8	0.0633	0.0855	0.0538	0.0896	0.0665	0.0843
9	0.0740	0.0648	0.0538	0.0861	0.1211	0.0976
10	0.1018	0.1658	0.0645	0.0850	0.1209	0.1054
11	0.0740	0.0648	0.0538	0.0287	0.0395	0.0898
12	0.1018	0.1658	0.0645	0.1194	0.1352	0.0976

Table 3 show the The results of fixed cost allocation plan based on the The presented algorithm.

Table 3. The results of fixed cost allocation plan.

DMUs	ρ_j	R_j^o	Fixed Cost Allocation
1	0.0765	7.6518	8.3333
2	0.0787	7.8668	8.3333
3	0.0817	8.1701	8.3333
4	0.0662	6.6184	7.7141
5	0.0714	7.1401	8.3333
6	0.1071	10.7052	8.3333
7	0.0820	8.2018	8.3333
8	0.0738	7.3835	8.0901
9	0.0829	8.2901	8.3333
10	0.1072	10.7235	7.7721
11	0.0584	5.8434	8.3136
12	0.1141	11.4052	9.7768
sum	-	100	100

5 | Conclusion

This paper proposes a DEA-based fixed cost allocation plan approach account. We consider undesirable outputs and propose fixed cost allocation plan by considering the the principle of full-efficient mechanism. We apply DDF model for obtain alternative fixed cost allocation plan. By choosing different direction vectors, we can flexibly change the fixed cost allocation plan and the cost allocated to the units will also change. The proposed fixed cost allocation plan allocated cost among efficient and inefficient units. We illustrate the results of the proposed approach with a numerical example. We showed that based on the presented approach we can obtain a fair cost allocation in the presence of undesirable outcomes for each of the DMUs. The proposed approach can also be developed based on other strategies in the fixed cost allocation plan, such as the principle of no change in efficiency. As future work, the models presented in this paper can be extended to the two-stage network structure.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

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