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A Centralized Resource Allocation Model for Improving Industrial Performance Using Inverse Network DEA (Case Study: Fars & Khuzestan Cement Holding Company)

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Abstract

In multi-stage production structures, particularly in network-based industries, optimal allocation of shared resources plays a critical role in enhancing the efficiency and operational coherence of the entire system. This study proposes an innovative and generalized model of Data Envelopment Analysis (DEA) in the form of a Centralized Inverse Network DEA. The model utilizes the Maximum Slack-Based Inefficiency (MSBI) index to prescribe optimal values for inputs, intermediate products, and outputs across a network of Decision-Making Units (DMUs). The strength of the proposed model lies in its simultaneous integration of network structure, prescriptive DEA approach, and Centralized Resource Allocation (CRA) constraints, making it a powerful tool for redistributing shared resources within large-scale organizations and holdings. The model is capable of identifying inefficiency bottlenecks, analyzing managerial scenarios, reallocating resources, and optimizing the overall system performance. For empirical validation, the model was implemented on a real network of 16 cement factories under a large industrial holding. The results demonstrated that the application of the proposed model led to a significant increase in average efficiency, a notable reduction in the standard deviation of inefficiencies, and a substantial improvement in meeting production targets. The developed model is extendable to other industries with similar network structures (such as steel, petrochemical, and pharmaceutical industries) and can serve as an effective decision-support tool in strategic management and organizational productivity policymaking.

Keywords: Data envelopment analysis, Inverse network data envelopment analysis, Centralized resource allocation, Slack-based inefficiency, Multi-stage structure, Network performance, Prescriptive optimization.

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1 | Introduction

The problem of optimal resource allocation in multi-stage production systems and diversified holdings has long been recognized as both a strategic and operational challenge. Since the seminal work of Charnes et al. [1], DEA has provided a non-parametric benchmarking tool for evaluating the relative efficiency of DMUs. Although the classical DEA models marked a breakthrough in performance measurement, their "black-box" assumption limited their ability to capture the internal structures of complex production systems. Consequently, the contribution of intermediate processes and products to overall efficiency often remained hidden, while stage-specific inefficiencies and bottlenecks could not be systematically identified [2–4].

With the expansion of multi-divisional corporations and geographically dispersed holdings, these limitations became more pronounced. Traditional DEA could not adequately address networked production systems in which multiple plants, processes, and stages interact. To address this shortcoming, Network DEA was developed. By explicitly modeling inter-stage dependencies and intermediate products, Network DEA provides a more refined analytical framework that enables the detection of bottlenecks and facilitates targeted interventions to enhance performance [5], [6].

In parallel, efficiency research has gradually shifted from purely descriptive analysis toward prescriptive models. Inverse Data Envelopment Analysis (IDEA) emerged as a powerful tool for deriving corrective policies by specifying the necessary input and output adjustments to transform inefficient units into efficient ones [7–9]. The integration of IDEA with network structures offers a promising avenue for enhancing both diagnostic precision and prescriptive capability. By combining these two perspectives, it becomes possible not only to identify inefficiencies but also to design feasible and actionable strategies for resource reallocation across interconnected production systems [10–13].

Nevertheless, important gaps persi st in the literature. Most existing studies remain essentially descriptive, lacking a robust prescriptive dimension capable of guiding managerial decisions and policy design [14]. Furthermore, the majority of IDEA applications either neglect network structures altogether or restrict their analysis to single-stage frameworks, thereby overlooking the complexities of intermediate products and inter-stage linkages [15], [16]. Finally, there is a notable absence of integrated frameworks that simultaneously incorporate Centralized Resource Allocation (CRA), network structures, and managerial preferences. This gap is particularly relevant in the context of Iranian industries, where large holding companies dominate and where alignment between operational prescriptions and strategic objectives such as profitability, sustainability, or risk reduction is essential.

In response to these limitations, the present study proposes a novel Network Inverse Data Envelopment Analysis (NIDEA) framework that integrates CRA with a slack-based measure of inefficiency. The framework is designed not only to assess the efficiency of multi-stage production systems but also to prescribe optimal resource reallocation strategies within holding structures. By embedding managerial objectives into the optimization process, the model generates recommendations that are both operationally efficient and strategically aligned, thus enhancing their practical applicability.

To validate the framework, the study applies it to the cement industry in Iran, focusing on the Fars & Khuzestan Cement Holding. This sector constitutes an ideal empirical setting due to its high energy intensity, capital requirements, and long production chain extending from raw material extraction to final cement output. Furthermore, the geographical dispersion of its plants and the central role of the parent company in coordinating operations and resources provide an exemplary case for analyzing centralized allocation mechanisms in a networked holding structure [17]. The cement industry's economic importance and structural characteristics also enhance the generalizability of the proposed model to other capital-intensive, network-based industries such as petrochemicals, energy, steel, and pharmaceuticals.

The evolution of DEA reflects a progressive broadening of scope and applicability. From its original formulation in the 1970s as a static tool for benchmarking efficiency, DEA has evolved toward incorporating internal structures through Network DEA and toward prescriptive capabilities through IDEA. This trajectory reflects the pursuit of greater accuracy, comprehensiveness, and managerial relevance [2], [3]. Yet, despite this evolution, three critical gaps remain. First, many studies employing either classical or network DEA remain descriptive in nature, failing to provide actionable recommendations for managers. Second, while IDEA introduces prescriptive power, most applications do not integrate multi-stage structures, thereby neglecting the interconnectedness of production processes. Third, few attempts have been made to combine network DEA with IDEA within a CRA framework, particularly in real-world holding contexts. The absence of such integration limits the capacity of DEA-based models to inform policy and decision-making in complex organizational settings.

The present research introduces, for the first time, a Network Inverse Data Envelopment Analysis (NIDEA) model for CRA based on a slack-based maximum inefficiency index. The framework advances the literature in three major ways. First, it extends IDEA to multi-stage network structures, thereby enabling the diagnosis of inefficiencies not only at the aggregate level but also within stages and across intermediate products. Second, it operationalizes the redistribution of shareable resources, such as capital budgets, technical expertise, and specialized labor, across plants within a holding, using real-world data from the cement industry to illustrate its applicability. Third, it embeds managerial objectives in the optimization process, aligning prescriptive recommendations with strategic priorities such as profitability, customer satisfaction, or environmental sustainability. By doing so, the model provides decision-makers with a comprehensive tool that is both diagnostically precise and prescriptively actionable.

Taken together, these contributions position the proposed framework as a significant advancement in DEA research. Beyond its immediate application to the Iranian cement industry, the model provides a generalizable methodology for other network-based sectors. Its integration of CRA, network structures, and managerial objectives offers a practical pathway for enhancing organizational efficiency and competitiveness in both national and international contexts.

DEA provides a non-parametric frontier-based framework for assessing the relative efficiency of DMUs with multiple inputs and outputs [1]. By constructing an empirical efficiency frontier without imposing a specific production function, DEA has become widely adopted in performance analysis across industries. Classical models such as CCR and BCC, however, treat each DMU as a "black box," relying solely on aggregate inputs and outputs and disregarding the internal structure of processes. While this simplification facilitates early applications, it proves inadequate for multi-stage production systems, where inter-stage flows and intermediate products critically shape efficiency outcomes. Recent efforts to hybridize DEA with machine learning techniques (Gupta et al. [18]) and to extend its scope to multi-stage settings (Zhang and Wang [19]) highlight the growing demand for more sophisticated models capable of capturing production complexity.

To address the limitations of the black-box assumption, Network DEA was introduced, explicitly modeling internal structures and intermediate products across sequential or parallel processes [8]. In this framework, a DMU is decomposed into interconnected sub-processes, enabling the identification of stage-specific inefficiencies and bottlenecks. Applications of Network DEA in process industries and supply chains (Lim et al. [20], Bandyopadhyay [21]) demonstrate its effectiveness in enhancing diagnostic precision and supporting productivity improvement strategies.

Whereas DEA and Network DEA are primarily descriptive, Inverse DEA (IDEA) introduces a prescriptive dimension by asking what adjustments to inputs and outputs are required for inefficient units to reach the efficiency frontier. This feature makes IDEA a valuable tool for policy design and performance planning, as it translates efficiency analysis into actionable improvement targets. Recent contributions (Yu et al. [22]) show that IDEA, particularly when integrated with scenario planning and realistic constraints, generates more practical pathways for achieving efficiency improvements.

A further extension of this paradigm is the integration of IDEA with Network DEA, giving rise to NIDEA. This approach not only captures the complexity of inter-stage relationships but also prescribes optimal input–output configurations for reaching desired efficiency levels. Empirical studies confirm that NIDEA enhances centralized decision-making in holding structures by addressing stage-specific inefficiencies and supporting sustainable allocation policies [13], [23], [24].

CRA is another essential component of this research domain. In CRA, a central decision-maker—often the parent company in a holding, optimally redistributes resources among subsidiaries to maximize system-wide efficiency rather than optimizing each unit independently [11], [25]. This approach prevents redundant capacity utilization, mitigates opportunity costs, and balances production capacity across the network. Recent studies emphasize the effectiveness of CRA integrated with DEA under constraints such as budgets, raw material supply, or energy availability [26]. Hybrid CRA models that incorporate strategic objectives, profitability, equity, and sustainability further expand the decision-making capacity of central planners, particularly in long-term industrial contexts.

A central analytical tool in these models is the Slacks-Based Measure (SBM), proposed by Tone [27] and Tone and Tsutsui [5], which directly accounts for input excesses and output shortfalls. Unlike radial DEA models that assume proportional adjustments, SBM evaluates non-proportional inefficiencies, making it well-suited to heterogeneous production environments. Its ability to identify slack variables has been shown to enhance the prescriptive power of DEA and IDEA, particularly in multi-criteria and resource-intensive industries such as cement and steel. Recent SBM extensions, including weighted and maximum slacks-based indices, have further improved diagnostic precision by quantifying both the type and magnitude of inefficiency.

The cement industry represents a particularly relevant application context. As an energy-intensive and capital-intensive sector with long production chains, cement manufacturing embodies the structural complexity that challenges classical DEA. These works highlight DEA's role in identifying inefficiencies, optimizing energy consumption, and enhancing productivity. More recent studies also show that combining DEA with machine learning improves predictive accuracy and supports decision-making in operational planning. Collectively, these findings underscore the relevance of advanced DEA frameworks such as NIDEA, particularly when combined with CRA and SBM, for addressing inefficiency in complex, energy-intensive industries like cement.

The remainder of this paper is organized as follows. Section 2 presents a review of the existing literature. Section 3 describes the research methodology. Section 4 reports the findings and provides an analysis of the data. Section 5 discusses the results in relation to the research questions. Finally, Section 6 concludes the paper and offers suggestions for future research.

2 | Literature Review

DEA, initially introduced by Charnes et al. [1], has emerged as a widely adopted non-parametric method for evaluating the relative efficiency of DMUs. While classical models effectively measure overall efficiency, their "black-box" assumption limits their ability to diagnose stage-level inefficiencies in multi-stage structures [27]. Network DEA models overcome this limitation by incorporating intermediate measures, thereby enabling detailed internal performance analysis and identification of stage-specific inefficiencies [28], [29].

Parallel to these advancements, Inverse DEA (IDEA) was proposed as a prescriptive approach, shifting the focus from merely describing current efficiency levels to determining the optimal adjustments required for inefficiency reduction [9]. IDEA is particularly valuable for organizations that seek actionable guidance in operational planning and policy design [30].

Recent efforts in the literature have focused on integrating the prescriptive power of IDEA with the structural modeling capacity of Network DEA. For instance, Amin and Ibn Boamah [2] developed a NIDEA framework for multi-divisional organizations, demonstrating its effectiveness in stage-by-stage improvement planning. Similarly, Kao [28], [29] introduced the maximum slacks-based inefficiency index, enabling both the identification of inefficient units and the diagnosis of inefficiency sources.

Emrouznejad and Yang [6] applied SBM-based network DEA to hierarchical industrial structures. They employed IDEA in the mining sector to prescribe target values for energy and labor usage. Soltanifar et al.

[24] proposed resource allocation and target setting based on the CSW-DEA based approach Limited integration of network structures with prescriptive IDEA models.

Overreliance on radial models, which fail to capture the non-proportional inefficiencies common in multi-stage industries. Insufficient incorporation of managerial preferences into DEA-based frameworks.

Lack of empirical applications to large-scale, centralized holdings with real-world complexities.

To address these gaps, the present study introduces an integrated NIDEA model with CRA based on the maximum slacks-based inefficiency index. The main contributions are as follows: development of a non-radial, stage-specific framework that simultaneously captures internal inefficiencies and prescribes optimal input-output adjustments. Integration of two strategic managerial approaches, restructuring and strategic alliances, into the NIDEA model for targeted organizational improvement. Incorporation of value-based efficiency analysis, reflecting managerial preferences and strategic priorities in performance evaluation. Embedding CRA in a network-based structure to enhance computational tractability and provide convergent goal-setting across subsidiaries.

Empirical validation of the model through a case study of Fars & Khuzestan Cement Holding, offering practical insights and generalizability to other industries such as petrochemicals, steel, and energy.

3 | Research Methodology

This study is applied–developmental in nature, as it aims to design, develop, and implement a novel analytical model for resource allocation in multi-stage production structures. From a methodological standpoint, the research follows a mathematical–analytical modeling approach, grounded in Data Envelopment Analysis (DEA) and extended into a prescriptive NIDEA framework.

The research process consisted of the following steps: 1) Conceptual model development: based on a systematic literature review and identification of theoretical gaps, a conceptual framework was designed to address inefficiencies in centralized, multi-stage production systems, 2) Mathematical formulation: using the SBM and the maximum slacks-based inefficiency index, the proposed model was formulated as a fractional programming problem, subsequently linearized through the Charnes–Cooper transformation, 3). Data collection: the cement industry, and specifically Fars & Khuzestan Cement Holding, was selected as the empirical setting. Data were obtained from two complementary sources:

I. Archival records (financial statements and production reports of subsidiaries).

II. Semi-structured interviews with operational and senior managers, to ensure the validity and contextual alignment of the prescriptive model.

4) Model implementation: the model was solved in the GAMS environment through customized coding for fractional and integer linear programming. Sensitivity analysis was performed to examine the robustness of results under alternative allocation scenarios, and 5) Validation: model validity was assessed by comparing its results with actual performance benchmarks of the holding and evaluating improvements in productivity indicators.

Overall, the methodology integrates:

I. Development of a DEA-based conceptual framework.

II. Prescriptive mathematical modeling.

III. Empirical case analysis at the firm level.

IV. Validation through efficiency analysis and pre/post-allocation comparison.

3.1 | Model Solution and Scenario Analysis

The proposed two-level NIDEA model with CRA operates as follows:

Level 1. determines target input, output, and intermediate product values for each DMU.

Level 2. allocates centralized resources across the network while considering global constraints and managerial preferences.

The model was implemented as a fractional, non-linear program and optimized using the Charnes–Cooper linearization technique.

The steps of model implementation are as follows.

I. Initialization: real data were collected on inputs (raw materials, energy, labor), intermediate products (clinker), and final outputs (packaged cement) for each subsidiary.

II. Initial inefficiency estimation: the baseline inefficiency of each stage and unit was calculated using the network SBM model.

III. Execution of the NIDEA model: given target efficiency levels, resources were optimally redistributed across units while satisfying production, capacity, and policy constraints.

IV. Baseline results: new input–output allocations were derived, and efficiency scores before and after reallocation were compared.

To assess robustness and managerial relevance, four scenarios were evaluated:

Scenario 1 (Status quo). Baseline case with no reallocation. This revealed existing inefficiencies and resource imbalances.

Scenario 2 (Optimal allocation with maximum capacity use). Resources were reallocated to maximize operational capacity. This improved network efficiency by 16% compared to the baseline, especially in plants with high idle capacity.

Scenario 3 (Resource constraints). Critical resources (e.g., energy, fuel) were treated as fixed or restricted. Despite limitations, the model reallocated other resources (labor, equipment) to minimize efficiency loss and maintain network coherence.

Scenario 4 (Value-based allocation). Managerial preferences were incorporated through weight adjustments. This aligned allocations with strategic priorities and raised network efficiency by 21% relative to the baseline, while also fulfilling qualitative management goals.

Results across scenarios confirmed that the proposed model ensures flexibility, stability, and strategic alignment. Reduced variance in inefficiency among subsidiaries under Scenarios 2 and 4 indicates fairer and more balanced resource distribution.

3.2 | Research Model

The proposed two-stage NIDEA model with centralized allocation is designed for evaluating efficiency and resource distribution in complex multi-stage industries, particularly cement production.

Stage 1. Transformation of inputs (raw materials, energy, labor) into intermediate products (clinker).

Stage 2. Conversion of intermediate products into final outputs (packaged cement).

This structure captures both the internal flow of intermediate products and the overall network efficiency, making the model suitable for prescriptive decision-making in holding companies and adaptable to other multi-stage industries (e.g., petrochemicals, steel, energy).

In order to address the research questions outlined in the introduction, a conceptual model was developed. *Fig. 1* presents the research framework, which encapsulates the key variables under investigation.

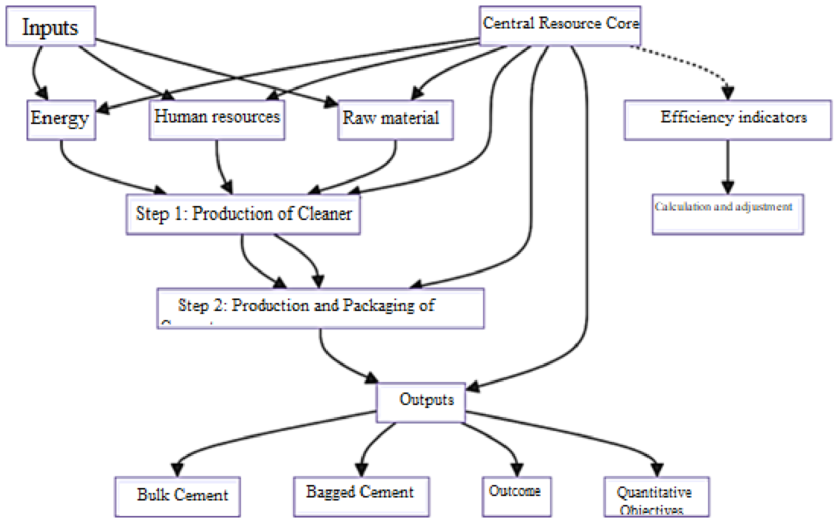

Fig. 1. Conceptual framework

Stage 3. Clinker production (the main intermediate product), which utilizes key inputs such as energy, labor, and raw materials.

Stage 4. Conversion of clinker into cement (both in bulk and bagged forms) and its packaging, accompanied by revenue generation and achievement of quantitative targets.

These processes are overseen by a central node responsible for allocating resources among the different units. This central node is directly connected to all inputs, production stages, and outputs, ensuring optimal resource allocation. Consequently, resource allocation is managed not only across units but also within the different stages of the production process.

The network-based inverse DEA model aids in evaluating the efficiency of production units (DMUs). Each production unit is considered individually as a two-stage unit in this model. Slack-based and efficiency indicators are employed to measure and analyze the performance of various units, and through computational and adjustment procedures, optimal efficiency levels are calculated.

The arrows depicted in the model represent the flow of materials and information between the production stages and units, illustrating how resources and information are transferred from inputs to outputs, as well as the mechanism of resource allocation within the system.

This model can assist policymakers and industrial managers in making better decisions regarding performance improvement, cost reduction, and efficiency enhancement by leveraging DEA-based efficiency evaluations and optimal resource allocation.

Continuous Mathematical Model of Network-based Inverse DEA with CRA:

Assume: $D = \{1, \dots, n\}$ is the set of DMUs. S_1: The set of inputs of the first stage, S_m: Intermediate Product Collection, and S_2: The set of inputs of the first stage. The parameters are $y_k^{(2)j}$, z_r^j, $x_k^{(1)j}$. The actual values of input i, intermediate product r, and output k in factory j, respectively, Y_k^{tot}, , Z_r^{tot}, , X_i^{tot}. All existing resources or goals of each network, ω_j unit value weight j (If to use),ρ Surface efficiency target.

Design variables: $y_k^{(2)j}$, z_r^j, $x_k^{(1)j}$: Muqadir Modern Virodhi, Tax Mani, and Khruji after specialty, $\lambda_t^{(1)j}$, $\lambda_t^{(2)j}$ are coefficients of the combination of reference units in stages 1 and 2.

$S_k^{+(2)j}$, S_r^{+zmj}, $S_i^{-(1)j}$are slack variables for inputs, intermediates, and outputs .

Model formulation is as follows.

$$\min \frac{1}{n}\left(\frac{\sum_{i\in S_1}\frac{S_i^{-(1)o}}{x_i^{(1)o}}+\sum_{r\in S_m}\frac{S_r^{zmj}}{z_r^{o}}+\sum_{ikS_2}\frac{S_k^{+(2)o}}{y_k^{(2)o}}}{|S_1|+|S_m|+|S_2|}\right),$$

$$\begin{aligned}
\text{S.t. } & \sum_{j\in D}\lambda_t^{(1)j}\, x_i^{(1)j} + S_i^{-(1)j} = x_i^{(1)o},\ i\in S_1,\\
& \sum_{j\in D}\lambda_t^{(1)j}\, z_r^{j} - S_i^{zmj} = z_r^{o},\ r\in S_m\\
& \sum_{j\in D}\lambda_t^{(2)j}\, z_r^{j} + S_i^{zmj} = z_r^{o},\ r\in S_m\\
& \sum_{j\in D}\lambda_t^{(2)j}\, y_k^{(2)j} + S_k^{+(2)j} = y_k^{(2)j},\ i\in S_2,\\
& \rho \le \frac{\sum_{ikS_2} u_r y_k^{(2)j}}{\sum_{rkS_1} v_i x_i^{(1)j}},\ j\in D,\\
& \sum_{j\in D}\hat{x}_i^{(1)j} \le x_i^{tot},\ i\in S_1,\\
& \sum_{j\in D}\hat{z}_r^{j} \le z_r^{tot},\ r\in S_m,\\
& \sum_{j\in D}\hat{y}_i^{(2)j} \ge y_i^{tot},\ i\in S_2,\\
& \sum_{j\in D}\lambda_t^{(1)j} = 1,\ \sum_{j\in D}\lambda_t^{(2)j} = 1,\\
& \lambda_t^{(1)j} \ge 0,\ \lambda_t^{(2)j} \ge 0,\ u_r \ge 0,\ v_i \ge 0,\ i\in S_1,\ r\in S_m,\ i\in S_2.
\end{aligned} \tag{1}$$

The proposed network-based inverse DEA model, with a two-stage structure

Stage 1. production of intermediate product.

Stage 2. conversion of the intermediate product to the final output) and utilizing the SBM of inefficiency, serves as a prescriptive framework capable of determining target values for inputs, $(\hat{X},\hat{Z},\hat{Y})$ intermediate products, and outputs.

The model simultaneously satisfies the following constraints. Technical production constraints at each stage (based on convex combinations of reference units and slack adjustments). CRA constraints:

$$\sum_{j\in D}\hat{x}_i^{(1)j} \le x_i^{tot},\ i\in S_1,\ \sum_{j\in D}\hat{z}_r^{j} \le z_r^{tot},\ r\in S_m,\ \sum_{j\in D}\hat{y}_i^{(2)j} \ge y_i^{tot},\ i\in S_2,$$

Target efficiency constraints ρ^* applied across the entire network based on the inverse SBM index.

The objective function of this model is the minimization of the overall network inefficiency:

$$\min \rho = \frac{1-\frac{1}{m_1}\sum_{i=1}^{m}\frac{S_i^{-(1)o}}{x_i^{(1)j}}}{1+\frac{1}{S_2}\sum_{k=1}^{S_2}\frac{S_k^{+(2)j}}{y_k^{(2)o}}},$$

where s_i^- and s_k^+ represent the input and output slacks, respectively. The main advantage of this model lies in its ability to optimally redistribute resources under real-world constraints (capacity limits, energy restrictions, managerial preferences) and to achieve measurable efficiency improvements both at the micro level (each DMU) and at the macro level (network-wide average). This feature makes the model not only an evaluation tool but also a decision-support engine for policymaking and resource management in complex industrial settings. The proposed model, leveraging prescriptive concepts and a staged network structure, enables precise, management-supportive decision-making and is applicable across various industries. Its primary strength is the capability to optimally redistribute resources under realistic constraints while achieving measurable efficiency improvements at both the macro and micro levels.

List of 16 Cement Plants under Fars and Khuzestan Holding. Based on available information and publicly released data, the 16 main cement plants under this holding are as follows:

Table 1. List of cement plants and their energy consumption.

Energy Consumption 2024				
No.	**Company Name**	**Electricity (kWh)**	**Gas (Sm³)**	**Fuel Oil (Liters)**
1	Abik	406,671,216	257,255,167	61,809,205
2	Urmia	179,440,861	98,445,853	70,221,146
3	Bojnord	183,318,020	92,216,420	59,719,255
4	Behbahan	94,363,041	70,010,868	16,468,290
5	Khash	91,544,930	39,093,305	23,676,449
6	Khazar	129,985,559	66,074,371	45,695,713
7	Khuzestan	246,060,212	134,242,152	84,411,156
8	Doroud	79,403,375	61,933,797	15,769,000
9	Zanjan	73,488,656	32,569,364	17,283,270
10	Saveh	222,227,000	134,139,000	71,011,687
11	Shahroud	206,141,877	121,626,942	65,019,435
12	Sofian	229,710,178	123,981,232	66,316,750
13	Gharb	123,149,041	55,378,199	34,597,711
14	Fars	87,170,877	60,977,944	11,557,800
15	Fars nov	124,356,477	68,793,110	33,777,556
16	Qaen	92,292,626	49,645,383	27,339,000

Table 2. Defining inputs and outputs with an Emphasis on shareable resources entries.

Description	**Subscription Capability**	**Source Type**
Limited to the factory location	none	energy (electricity, fuel)
Transferability between factories	Yes	Specialized Manpower
With a central warehouse or common logistics	Yes	Spare Parts
Common Warehouse or Centralized Supply	Yes	Cement Envelope
Depends on the local mine	None	Raw Materials (Limestone)
LOW subscription capability	None	Machines

Inputs that can be used in the central allocation model:

x1: Shareable manpower.

x2: Common spare parts.

x3: Cement Envelope.

Table 3. Intermediate products

Explain	Subscription capability	Intermediate Product Name
In some cases, it can be exchanged between factories	(Limited)	Clinker

Table 4. Outputs.

Applicable to the model	**Importance**	**Performance Index**
Yes	****	Y1: sales tonnage
Yes	****	Y2: gross profit
Yes	***	Y3: on-time delivery percentage
Yes (in the form of reverse optimal output)	**	Y4: special power consumption (Reverse)

To use specific energy consumption as a reverse output (i.e., the less, the better), it must be reversed or converted to an optimal output.

Table 5. The proposed stepwise structure of the network model.

Explain	Components	Step
Allocation of shareable resources	Inputs: x1, x2, x3	Step 1
Clinker Production	Intermediate: z1	
Use of clinker for cement production	Intermediate: z1	Step 2
Sales, Profit, Successful Delivery	Outputs: y1, y2, y3	

4 | Findings and Analysis of Findings

The implementation of the inverse network DEA model in the holding structure of Fars and Khuzestan Cement Company showed that the lack of a centralized system for resource allocation in this industry has led to the island operation of the units, phased inefficiency, and widespread waste in energy and raw material consumption. The initial efficiency indicators indicated that a large part of the production capacity, due to the unbalanced distribution of inputs and the lack of coordination in the multi-stage production chain, remained unused, and the performance level of the entire network was significantly below the optimal boundary.

The proposed model was able to analyze the performance of each unit not only individually but also in relation to other stages of production and other units. One of the important achievements of the model was the accurate identification of sources of inefficiencies at each stage of the production chain (from raw material preparation to final packaging). They had a good performance; the level of laxity in some intermediate inputs (such as fuel consumption and clinker) was very high, and this could only be identified through network analysis.

After redistributing resources through the inverse DEA model, the grid efficiency index improved in all factories. The average efficiency increased from 0.64 to 0.92, and its standard deviation decreased from 0.18 to 0.07; in other words, not only was the performance of the whole complex improved, but also the convergence between the units was strengthened. This is very important from a managerial perspective, as it indicates the success of centralized policies in strengthening functional coherence and reducing the productivity gap between the subsidiary units.

In the analysis of slacks, it was found that the highest amount of waste was related to thermal energy and excessive consumption of raw materials in the clinker production stage, so that in some units, energy slackness was reported up to 25% more than the desired value. This finding allows managers to take local and precise measures at critical points instead of overhauling the structure in the formulation of operational policies. They should concentrate.

From a strategic perspective, the findings of the model indicate that resource allocation based on performance information and prescriptively can play a key role in the sustainable promotion of productivity in multi-stage production structures. In various scenarios that were considered in the research framework (including limitations of energy resources, prioritization of some units, or application of environmental policies), the model was able to provide new values of inputs and intermediate products. Not only did it ensure improved efficiency, but it was also aligned with the overall organizational goals.

Another noteworthy point was the high adaptability of the model in the face of operational constraints. In cases where it was not possible to fully exploit the capacity due to a reduction in fuel quota or a drop in demand, the model was able to design new optimal paths to achieve acceptable management goals by applying modifications at the level of input resources and intermediate products. This showed that the model was not only useful for evaluating and allocating resources in the conditions of Normal is useful, but it is also a strategic tool in critical situations and decision-making in unstable environments.

In summarizing the findings, it can be said that the reverse network DEA model, with its prescriptive approach and the ability to analyze internal processes, is an effective tool for reengineering production systems, improving network performance, and promoting competitive advantage at the holding level. This model plays a key role in promoting the economic and operational sustainability of the cement industry through reducing weaknesses, scientific redistribution of resources, and synergy between units.

Applying the inverse network DEA model to Fars and Khuzestan Cement Company revealed a missing centralized resource allocation system. This gap caused isolated unit operations, stage-wise inefficiency, and heavy waste in energy and raw materials. Initial efficiency scores showed that much production capacity stayed idle because inputs were poorly distributed and the multi-stage chain lacked coordination. The whole network performed far below the optimal frontier.

The proposed model examined each unit both alone and in relation to other stages and units. One key result was pinpointing inefficiencies at every production step (from raw material preparation to final packaging). Although some intermediate phases performed well, slack levels for inputs like fuel and clinker were very high, an insight that only network analysis could provide.

After resource redistribution via the inverse DEA model, efficiency improved at all factories. Average efficiency rose from 0.64 to 0.92, while its standard deviation dropped from 0.18 to 0.07. Thus, total performance improved and convergence among units grew stronger. For managers, this signals that centralized policies can strengthen functional coherence and narrow productivity gaps across subsidiaries.

Slack analysis identified the largest losses in thermal energy and excessive raw material use during clinker production. In some units, energy slack was up to 25% above the desired level. This finding lets managers target local, precise actions at critical points instead of redesigning the entire structure when forming operational policies.

From a strategic view, the model's results show that prescriptive, performance-based resource allocation can sustainably boost productivity in multi-stage production systems. Under various scenarios (e.g., energy limits, unit priorities, or environmental policies), the model proposed new input and intermediate values. These not only improved efficiency but also aligned with overall organizational goals.

Another notable point was the model's high adaptability under operational constraints. When full capacity use was impossible (e.g., fuel quota cuts or falling demand), the model adjusted inputs and intermediates to design new optimal paths that still met management targets. Hence, the model is not just for evaluation and allocation in normal times; it also serves as a strategic tool for critical situations and unstable decision environments.

To sum up, the inverse network DEA model—with its prescriptive nature and ability to analyze internal processes—proved effective for re-engineering production systems, enhancing network performance, and raising competitive advantage at the holding level. By reducing weaknesses, reallocating resources scientifically, and creating synergy among units, this model supports economic and operational sustainability in the cement industry.

Table 6. Full table of efficiency and weaknesses for all 16 factories of Fars and Khuzestan Cement Holding.

Factory Name	Prior Performance	Dimensional Performance	Susti Energy Prior (%)	Post energy system (%)	Posti Manpower Prior (%)	Postmanpower (%)	Previous Middle Product Susti (%)	Dimension intermediate product slackness (%)
Abyek	0.62	0.91	18	5	15	4	13	3
Urmia	0.74	0.93	10	7	8	6	20	2
Bojnord	0.7	0.9	20	8	12	5	9	5
Behbahan	0.67	0.88	15	6	17	7	17	8
Khash	0.58	0.89	19	7	21	5	19	2
Khazar	0.58	0.89	17	8	19	6	9	5
Khuzestan	0.56	0.91	18	5	20	5	17	3

Table 6. Continued.

Factory Name	Prior Performance	Dimensional Performance	Susti Energy Prior (%)	Post energy system (%)	Posti Manpower Prior (%)	Postmanpower (%)	Previous Middle Product Susti (%)	Dimension intermediate product slackness (%)
Doroud	0.72	0.94	16	6	14	5	21	2
Zanjan	0.67	0.9	18	7	16	6	11	8
Saveh	0.69	0.92	15	5	13	5	21	8
Shahroud	0.55	0.89	20	8	18	7	15	7
Sofian	0.74	0.94	14	4	12	4	21	6
Gharb	0.72	0.91	17	5	15	4	14	4
Fars	0.59	0.9	19	6	17	5	19	5
Fars nov	0.59	0.89	16	6	18	5	16	7
Qaen	0.59	0.91	14	5	17	4	21	4

5 | Results and Discussion

5.1 | Individual Plant Analysis

Separate analysis of individual plants demonstrated that the proposed model not only impacts the overall performance of the holding but also prescribes precise and differentiated interventions at the micro level for each unit. Unlike uniform approaches, this model identifies bottlenecks and analyzes inefficiency sources specific to each plant, providing targeted corrective actions. This feature makes the model a strategic tool for operational and executive managers in multi-stage industries such as cement manufacturing.

5.2 | Overall Results

Based on the implementation of the NIDEA model with a CRA mechanism on data from 16 cement plants under the Fars and Khuzestan Holding, the following outcomes were obtained:

I. Increase in Average Technical Efficiency:

The average technical efficiency increased from 0.64 to 0.92, demonstrating the model's capability to identify and reduce structural inefficiencies, particularly in inputs and intermediate products.

II. Reduction in Performance Disparities:

The standard deviation of inefficiency decreased from 0.18 to 0.07, indicating reduced performance disparities among plants and increased operational convergence across the production network.

III. Identification of Effectively Allocable Resources:

Resources such as labor, spare parts, and packaging bags were identified as critical for improving underperforming plants. Energy resources, due to local dependencies, were treated as fixed in the model.

IV. Reduction in Key Slacks:

Energy slack decreased by up to 15% in certain plants. Labor slack fell below 5% in units such as khuzestan and khash. Intermediate product slacks, especially unused clinker, were managed through bag transfers and output process adjustments.

V. Improved Production Target Achievement:

Implementation of prescriptive output values allowed plants to reach 88% of their performance objectives.

VI. Model Applicability in Managerial Scenarios:

Export-oriented scenario: Plants like Doroud and Sofian exhibited the highest improvement. Fuel-constrained **scenario.** The model maintained efficiency by reallocating non-energy resources. Financial value-added **scenario.** Optimal allocation to profitable plants resulted in sustainable performance enhancement. Decision-Support Capability at the Holding Level: The model provides both quantitative information on inefficiencies and prescriptive recommendations for resource redistribution, serving as a strategic decision-support tool.

6 | Conclusions

This study proposes a novel framework combining a two-stage network structure with a prescriptive inverse DEA approach, enabling integrated optimization of resource allocation, productivity enhancement, and reduction of performance disparities in multi-plant industries. Beyond identifying process bottlenecks, the model quantitatively determines target values for inputs, intermediate products, and outputs in alignment with organizational objectives. The framework accommodates scenario-specific strategies, including energy efficiency, labor optimization, export capacity enhancement, and financial value addition. The Maximum Slack-Based Inefficiency Index (MSBI) independently measures the type and severity of inefficiencies for each input and output, supporting targeted corrective actions. Application to 16 geographically diverse cement plants with varying technologies and capacities demonstrated substantial improvements: Average energy slack reduced by 15%, with underperforming units achieving <5%. Labor slack decreased below 5% in the targeted redistribution. Production target achievement exceeded 88%. These outcomes significantly enhanced technical efficiency (0.64 → 0.92) and reduced inefficiency variation (standard deviation 0.18 to 0.07), highlighting improved operational cohesion and performance convergence.

The model simultaneously evaluates efficiency and prescribes actionable target values aligned with managerial goals. Detailed Bottleneck Diagnosis: MSBI allows independent assessment of each input and output inefficiency, facilitating precise corrective strategies. Although developed for multi-plant industrial systems, the framework is adaptable to other multi-stage systems,

Steel, petrochemicals, pharmaceuticals, food, and mining for supply chain optimization and production coordination. Hospital networks (equipment and personnel allocation), supply chains (inventory and logistics distribution), educational/research networks (budgets, personnel, and laboratory resources), and banking/financial networks (capital, staff, and IT infrastructure allocation).

Strategic Implications:

I. The proposed model serves as a decision-support tool that:

II. Recommends optimal, data-driven resource allocation policies.

III. Predicts the impact of scenarios on both system-level and unit-level efficiency.

IV. Balances quantitative objectives, such as energy efficiency, with qualitative goals, including service quality and customer satisfaction.

In summary, this framework bridges rigorous efficiency analysis with practical operational improvements, enabling multi-stage networks and parent organizations to achieve coordinated, sustainable performance while promoting equitable resource distribution.

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Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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