Research Annals of Industrial and Systems Engineering

www.raise.reapress.com

Res. Ann. Ind. Syst. Eng. Vol. 3, No. 1 (2026) 56–64.

Paper Type: Original Article

Mathematical Modelling for the Vaccine Allocation Considering the Lockdown Policy on Retail Units

Nafiseh Shamsi Gamchi[1,*], Maryam Esmaeili[1]

[1] Department of Industrial Engineering, Faculty of Engineering, Alzahra University, Tehran, Iran; n.shamsi@alzahra.ac.ir; esmaeili_m@alzahra.ac.ir.

Citation:

Received: 20 June 2025 *Revised: 29 August 2025* *Accepted: 19 October 2025*	Shamsi Gamchi, N., & Esmaeili, M. (2026). Mathematical modelling for the vaccine allocation considering the lockdown policy on retail units. *Research Annals of Industrial and Systems Engineering, 3*(1), 56-64.

Abstract

This paper proposes a vaccination prioritization strategy aimed at minimizing the social and economic costs of lockdowns, using the COVID-19 pandemic as a case study within a Susceptible–Exposed–Infectious–Recovered (SEIR) epidemic model framework. It critiques the common approach of prioritizing based solely on age (excluding healthcare workers) due to limited decision-making time and resources, overlooking significant economic impacts. By including economically essential groups, such as retailers, in the prioritization strategy alongside age groups, the model aims to enhance vaccination effectiveness. The study also accounts for varying vaccine efficacies and supply constraints from multiple suppliers. Optimal control theory is used to determine the required number of vaccine doses, and the model's effectiveness is demonstrated through an illustrative example.

Keywords: Infectious disease, Susceptible–exposed–infectious–recovered epidemic model, Vaccine allocation, Parallel prioritization, Optimal control, Retailers.

1 | Introduction

This paper addresses the critical need for an effective vaccination prioritization strategy during the COVID-19 pandemic, which led to severe global health and economic consequences, including over 4.5 million deaths worldwide and 123,000 in Iran as of October 2021. Given the limited availability of effective vaccines [1] and the high costs of non-pharmaceutical interventions like lockdowns [2], prioritization strategies became essential. While the World Health Organization (WHO) emphasizes vaccinating high-risk groups to reduce mortality, maintain infrastructure, and limit social and economic disruptions [3], traditional approaches often ignore the dynamic nature of epidemics and the economic burden on specific sectors, such as retail. Motivated by these shortcomings, the authors propose a novel mathematical model using a parallel vaccination strategy that aims to minimize social costs, vaccine procurement costs, and economic losses due to business closures. The model is informed by prior research on epidemic control tools [4], [5], and vaccine effectiveness [6].

Corresponding Author: n.shamsi@alzahra.ac.ir
https://doi.org/10.22105/raise.v3i1.87

Numerical simulations validate the model's performance and offer actionable insights for policymakers dealing with similar future outbreaks.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, the notation, assumptions, and the developed model are presented. Section 4 describes the solution approach presented to solve the model. In Section 5, some numerical examples are provided whose computational results validate the developed model. Finally, the paper concludes in Section 6.

2 | Literature Review

Infectious diseases remain a major global health threat, with epidemics occurring when disease cases exceed expected norms [7]. Accurate modeling of these dynamics is vital for forecasting infection spread and informing public health interventions [8]. Numerous epidemic models have been developed to capture the transmission behavior of COVID-19. These include compartmental models incorporating symptomatic stages [9], age-structured models [10], and Susceptible–Exposed–Quarantined–Infectious–Recovered (SEQIR) and Susceptible–Exposed–Infectious–Recovered–Deceased (SEIRD) models accounting for quarantined or undetected individuals [11]. Some studies, like [12], integrate social distancing and vaccination in deterministic frameworks to evaluate control measures. Alongside disease modeling, vaccine allocation strategies are a critical research focus due to limited vaccine supplies. Models have been developed for prioritization based on criteria such as age, health, and occupation [13], [14], and to assess the impact of different targeting strategies [15]. However, most fail to fully integrate epidemic models with mathematical optimization frameworks to simultaneously address social, economic, and vaccine supply constraints. This study addresses this gap by proposing a novel integrated approach for vaccine distribution that considers dynamic disease progression and real-world limitations.

While prior research has developed various criteria and methods to prioritize individuals for vaccination, many studies overlook integrating epidemic dynamics into optimization models aimed at minimizing broader impacts. A key limitation of existing work is the lack of models that jointly consider disease behavior and vaccine distribution to optimize outcomes like social and economic costs. Addressing this gap, the current paper proposes a novel three-objective mathematical model that combines epidemic modeling with optimization techniques. The model aims to minimize 1) the total vaccine procurement cost from multiple suppliers, 2) the social cost from infections occurring before and after vaccination, and 3) the economic losses due to shop closures during lockdowns. This integrated approach supports more effective, data-driven policymaking for vaccine allocation under resource constraints.

3 | Problem Description

In this paper, we investigate the impact of parallel vaccination among different priority groups on vaccine procurement cost, social cost caused by infected individuals of different priority groups, and the economic cost imposed on the business owners, shopkeepers, and retail stores during lockdown policy implementation. Therefore, we propose a mathematical model to determine the best vaccine allocation to the supposed priority groups and implement the lockdown policy and other health protocols. The health workers at high to very high risk of acquiring and transmitting infection are excluded from the proposed model. The priority groups are adults above 65 years old, individuals between 50 and 64 years old, and retailers. The retailers could start their activity after vaccination, even during the lockdown policy implementation. The vaccination has the herd effect, which refers to the indirect protection of unvaccinated people [16]. The proposed epidemic model includes susceptible, exposed, symptomatic, asymptomatic infected, hospitalized, home quarantine, isolated, and recovered individuals (*Fig. 1* and *Fig. 2*).

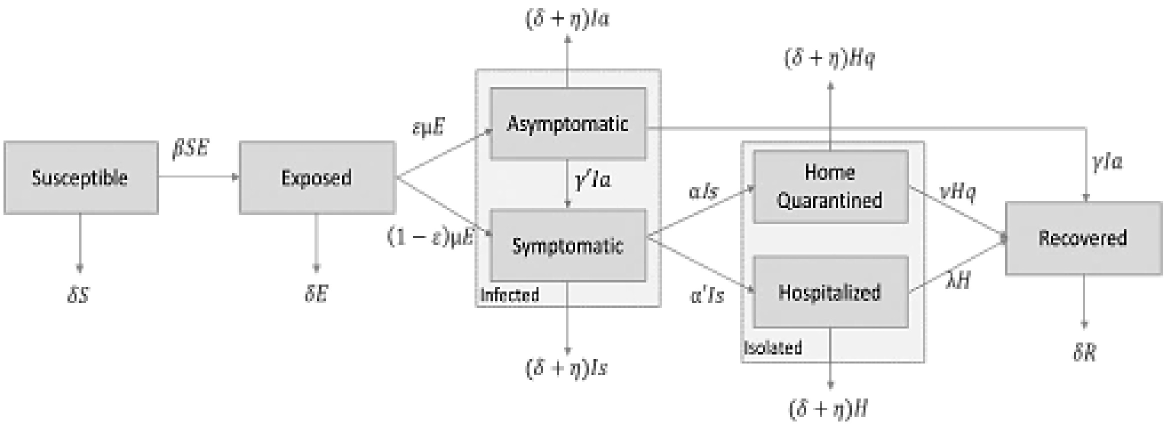

Fig. 1. Proposed Susceptible–Exposed–Infectious–Recovered (SEIR) epidemic model of COVID-19 before vaccination.

As shown in *Fig. 1*, when susceptible people are in contact with the exposed ones, with a determined rate of transmission, they could be changed to exposed. Some of the exposed ones become infected. Although in the case of COVID-19, some people are asymptomatically infected, this could result in another person's infection. Thus, we should consider it in the epidemic model. In confronting the symptomatic infected individuals, two different options are recommended, including home quarantine or hospitalization. The last-mentioned states move to a recovered state at different recovery rates.

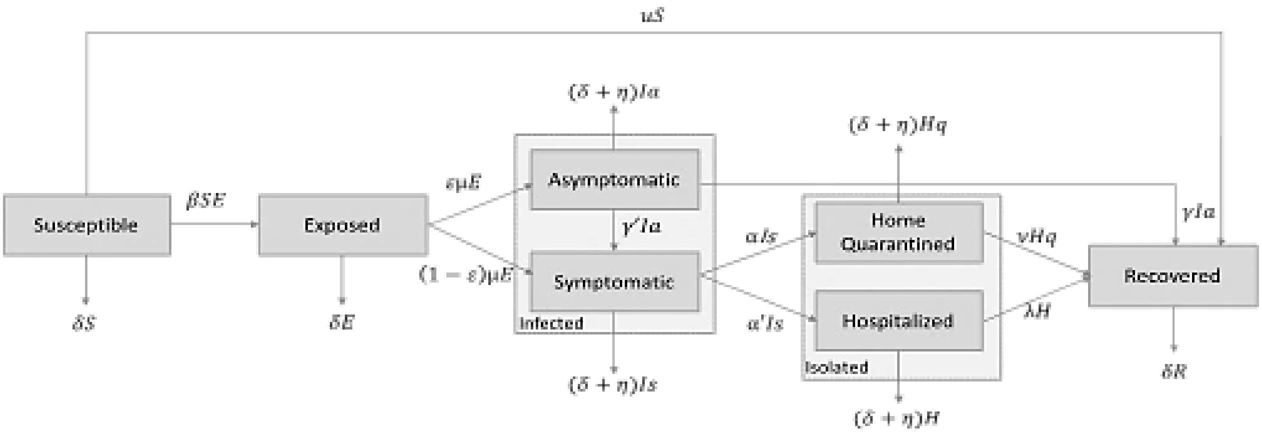

Fig. 2. Proposed SEIR epidemic model of COVID-19 after vaccination.

3.1 | Notations

i	Index of suppliers ($i = 1, \dots, n$)
j	Index of priority groups ($j = 1, \dots, m$)
k	Index of businesses that are affected by lockdown policy ($k = 1, \dots, P$)
p_i	Price of each vaccine dose from supplier i
Cap_i	Capacity of supplier i
BC_k	Cost of locking down business k
SC_j	Social cost caused by infected individuals of group j
$S_{1,j}(t)$ $(S_{2,j}(t))$	Number of susceptible individuals of the group j before vaccination (after vaccination) at time t
$E_{1,j}(t)$ $(E_{2,j}(t))$	Number of exposed individuals of the group j before vaccination (after vaccination) at time t
$Ia_{1,j}(t)$ $(Ia_{2,j}(t))$	Number of asymptomatic infected individuals of the group j before vaccination (after vaccination) at time t
$Is_{1,j}(t)$ $(Is_{2,j}(t))$	Number of symptomatic infected individuals of the group j before vaccination (after vaccination) at time t

$Hq_{1,j}(t)$ $\left(Hq_{2,j}(t)\right)$	Number of home-quarantined individuals of the group j before vaccination (after vaccination) at time t
$H_{1,j}(t)$ $\left(H_{2,j}(t)\right)$	Number of hospitalized individuals of the group j before vaccination (after vaccination) at time t
$R_{1,j}(t)$ $\left(R_{2,j}(t)\right)$	Number of recovered individuals of the group j before vaccination (after vaccination) at time t
ς_j	The recruitment rate of the priority group j
β_j	The disease transmission rate of the priority group j
δ_j	The natural death rate of the priority group j
η_j	The death rate from COVID-19 in the priority group j
μ_j	Progression rate of COVID-19 based on its incubation period in the priority group j
ε_j	The proportion of exposed individuals who have progressed into asymptomatic individuals of the priority group j
α_j	Rate of being home quarantined in the priority group j
α'_j	Rate of being hospitalized in priority group j
γ_j	Rate of natural recovery of asymptomatic infected individuals in priority group j
γ'_j	Rate of being symptomatic infected individuals from asymptomatic ones in priority group j
λ_j	Rate of recovery of hospitalized individuals in priority group j
$\mathcal{R}_{0,j}$	Basic reproduction number of the priority group j
T_j	The time at which the epidemic in the priority group j ends
C^v	Fixed cost of the immunization program
τ_j	Start time of the priority group j's vaccination
ϕ	Vaccine unit shortage cost
FC	The fixed unit cost of delivering a COVID-19 vaccine dose ($\frac{\$}{\text{dose.distance}}$)
d_{i0}	The distance between the supplier i and the central distribution depot

3.2 | Decision Models

The developed three-objective model aims to simultaneously minimize the vaccine procurement cost, the total social cost incurred by infected individuals of different priority groups (home quarantine or hospitalized ones) before and after vaccination, as well as the economic cost of shop closures. Therefore, finding out the best time to immunize each priority group and the number of vaccine doses assigned to each group is necessary.

The primary purpose of the first objective is to minimize the cost required to reduce or enhance the immunization intensity.

$$\text{Min } Z1 = \sum_i p_i . V_i + FC . d_{i0} . V_i . \quad (1)$$

Eq. (1) denotes the total cost of vaccine procurement from different suppliers. Since the vaccine price depends on the vaccine type and its brand, we consider different prices for the supplied vaccines. Also, we consider that the vaccines supplied by different suppliers have different effectiveness. In addition, in *Eq. (1)*, we consider the cost of vaccine delivery from the supplier to the central depot, which is shown in the second term.

$$\text{Min } Z2 = \sum_j [\int_0^{\tau_j} SC_j(Hq_{1,j}(t) + H_{1,j}(t))dt + \int_{\tau_j}^{T_j} (SC_j(Hq_{2,j}(t) + H_{2,j}(t)) + \frac{1}{2}C^v u_j^2(t))dt]. \quad (2)$$

The first term in *Eq. (2)* denotes the social cost of hospitalized and home-quarantined individuals of the priority group before vaccination. The second term is the cost, as mentioned earlier, after immunization, and the third is the vaccination cost considering the optimal variable. Notably, the third term aims to maximize the number of recovered individuals using the possible minimal control variable. Notably, the square of the control variable reflects the severity of vaccination side effects [17].

$$\text{Min Z3} = \sum_k [BC_k(\tau_J)(\int_0^{\tau_J} S_{1,J}(t)dt) + \int_{\tau_j}^{T_j}(BC_k(\tau_J)(Hq_{2,J}(t) + H_{2,J}(t))]. \tag{3}$$

The last objective function minimizes the economic impact caused by shop closures. The first term in *Eq. (3)* denotes the cost imposed on businesses due to the lockdown before the vaccination of retailers as a priority group. The second term is the cost imposed on the business due to the infection of retailers. The cost imposed on the business is a function of the period of the lockdown policy. For the sake of simplicity, we suppose a linear function for the mentioned cost as follows:

$$BC_k(\tau_J) = A\tau_J^2 + B. \tag{4}$$

Also, we consider a fixed cost of consumers losing to the shops. Thus, we would have *Eq. (4)* for the cost of business/shop closure.

Before the vaccination of each priority group, there is an epidemic model without a control tool, and the infected individuals recover naturally. The differential equations of the epidemic model would be as *Eqs. (5)-(11)*.

$$\frac{dS_{1,j}(t)}{dt} = \varsigma_j - \beta_j S_{1,j}(t)\sum_{j=1}^{m}\left(Ia_{1,j}(t) + Is_{1,j}(t)\right) - \delta_j S_{1,j}(t), \quad 0 \le t < \tau_j. \tag{5}$$

$$\frac{dE_{1,j}(t)}{dt} = \beta_j S_{1,j}(t)\sum_{j=1}^{m}(Ia_{1,j}(t) + Is_{1,j}(t)) - \mu_j E_{1,j}(t) - \delta_j E_{1,j}(t), \quad 0 \le t < \tau_j. \tag{6}$$

$$\frac{dIa_{1,j}(t)}{dt} = \varepsilon_j \mu_j E_{1,j}(t) - \gamma_j Ia_{1,j}(t) - \gamma'_j Ia_{1,j}(t) - (\delta_j + \eta_j) Ia_{1,j}(t), \quad 0 \le t < \tau_j. \tag{7}$$

$$\frac{dIs_{1,j}(t)}{dt} = (1 - \varepsilon_j)\mu_j E_{1,j}(t) + \gamma'_j Ia_{1,j}(t) - (\delta_j + \eta_j) Is_{1,j}(t) - \alpha_j Is_{1,j}(t) - \alpha'_j Is_{1,j}(t), \quad 0 \le t < \tau_j. \tag{8}$$

$$\frac{dHq_{1,j}(t)}{dt} = \alpha_j Is_{1,j}(t) - (\delta_j + \eta_j) Hq_{1,j}(t) - \nu_j Hq_{1,j}(t), \quad 0 \le t < \tau_j. \tag{9}$$

$$\frac{dH_{1,j}(t)}{dt} = \alpha'_j Is_{1,j}(t) - (\delta_j + \eta_j) H_{1,j}(t) - \lambda_j H_{1,j}(t), \quad 0 \le t < \tau_j. \tag{10}$$

$$\frac{dR_{1,j}(t)}{dt} = \nu_j Hq_{1,j}(t) + \lambda_j H_{1,j}(t) + \gamma_j Ia_{1,j}(t) - \delta_j R_{1,j}(t), \quad 0 \le t < \tau_j. \tag{11}$$

When the required dose of vaccine is assigned to each priority group and the immunization program is started, the epidemic model would change by considering the vaccination as a control tool.

Constraint ***Error! Reference source not found.*** indicates that we should supply the number of vaccine doses for all the priority groups to control the epidemic. Notably, the demand for vaccine doses is a function of the value of the optimal control tool, the number of susceptible individuals in the group, and the time of vaccination.

$$\sum_i V_i \ge \sum_j \int_{\tau_j}^{T_j} u_j^* S_{2,j}(t)dt. \tag{12}$$

Finally, *Eq.* ***Error! Reference source not found.*** denotes that the maximum number of vaccine doses supplied by supplier i equals its capacity.

$$V_i \le Cap_i. \tag{13}$$

4 | Solution Approach

The proposed model is designed to minimize both the social cost associated with infected individuals, specifically those who are hospitalized or in home quarantine, and the economic cost borne by retailers due to lockdown policies. Additionally, it addresses the challenge of limited vaccine availability by incorporating multiple vaccine suppliers, each offering different pricing, aiming to minimize overall vaccine procurement costs. Drawing on approaches from [6], [13], the model uses a two-phase solution method.

In Phase 1, the epidemic model is simulated to determine the total number of vaccine doses required. This process begins by calculating the basic reproduction number $\mathcal{R}_{0,j}$ for each priority group, using a nextgeneration matrix approach evaluated at the disease-free equilibrium. If $\mathcal{R}_{0,j} < 1$, the group is excluded from vaccination since the disease is unlikely to spread within that group. Otherwise, the simulation proceeds to identify the distribution of individuals across various compartments (e.g., susceptible, exposed, infected) over time.

$$\mathcal{R}_{0,j} = [\frac{\beta\varsigma\mu[(\delta+\eta+\gamma+\gamma')-\varepsilon(\alpha+\alpha'+\gamma+\gamma'+2\delta+2\eta)}{\delta(\mu+\delta)(\alpha+\alpha'+\delta+\eta)(\gamma+\gamma'+\delta+\eta)}]_j. \tag{14}$$

In Phase 2, optimal control theory is applied to determine the number of susceptible individuals in each priority group eligible for vaccination, calculate the necessary vaccine doses, and assess the evolving social cost of infections. The control variable $u_j^*(t)$ represents the vaccination rate, optimized using Pontryagin's Maximum Principle [17] by solving the associated Hamiltonian function. The final objective function is a weighted sum of the social and economic costs:

$$OBF = wt_1.Z2 + wt_2.Z3, \tag{15}$$

where wt_1 and wt_2 are the weights of objective functions 2 and 3, respectively, and $wt_1 + wt_2 = 1$.

5 | Computational Results

This section applies the proposed vaccination strategy to a real-world case study focused on Tehran, Iran. Three priority groups are analyzed: individuals aged 50–65 years (G1), those over 65 years (G2), and shopkeepers and retailers (G3). Based on national market data, Iran has approximately 5 million retail units, with 7% located in Tehran. Assuming an average of three active individuals per unit, the retailer group in Tehran is estimated to include about 1,050,000 individuals. These three distinct groups are used to assess the effectiveness and applicability of the model in a real urban setting.

The model highlights the importance of prioritizing shopkeepers and retailers to help reduce the economic impact of lockdowns. It offers a flexible framework for vaccine distribution that can be applied to various groups and epidemic scenarios. Since infection risks and social costs vary across groups, the model accounts for these differences—for example, older adults face higher infection risks, and each group incurs different social costs: $1,200 for ages 50–65, $1,000 for over 65, and $1,300 for retailers.

Table 1. Input parameters of the presented epidemic model.

Rate	Value (G1, G2, G3)
ς_j	0.52, 0.55, 0.57
β_j	0.60, 0.60, 0.60
δ_j	0.003, 0.006, 0.004
η_j	0.015, 0.03, 0.02
μ_j	0.53, 0.53, 0.53
ε_j	0.1, 0.1, 0.1
α_{1j}	0.73, 0.75, 0.72
α_{2j}	0.19, 0.21, 0.20
γ_{1j}	0.6, 0.6, 0.6
γ_{2j}	0.31, 0.34, 0.30
ν_j	0.9, 0.9, 0.9
λ_j	0.78, 0.7, 0.75

We also suppose a linear function of vaccination time for the cost of business lockdown (i.e., $BC(\tau_J) = A\tau_J + B$). In this case, we consider $A = 20\$$ and $B = 0\$$. Since in Iran, vaccination started about one year after the disease outbreak. Therefore, the dynamic of the epidemic would be as shown in *Fig.*

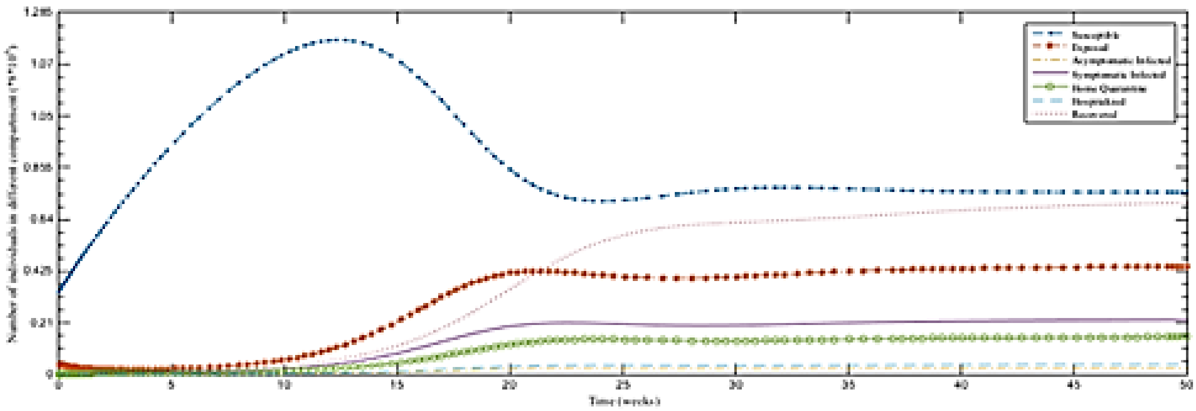

Fig. 3. Dynamics of the COVID-19 epidemic before vaccination.

As illustrated in *Figs. 3* and *4*, the number of susceptible individuals declines as exposed cases rise, and over time, many infected individuals (both home-quarantined and hospitalized) eventually recover. Once vaccination begins, the number of recovered individuals increases, while exposed and infected cases decrease significantly. Eventually, exposed and infected individuals drop to zero, though the number of recovered individuals gradually declines over time due to changes in population dynamics or immunity levels.

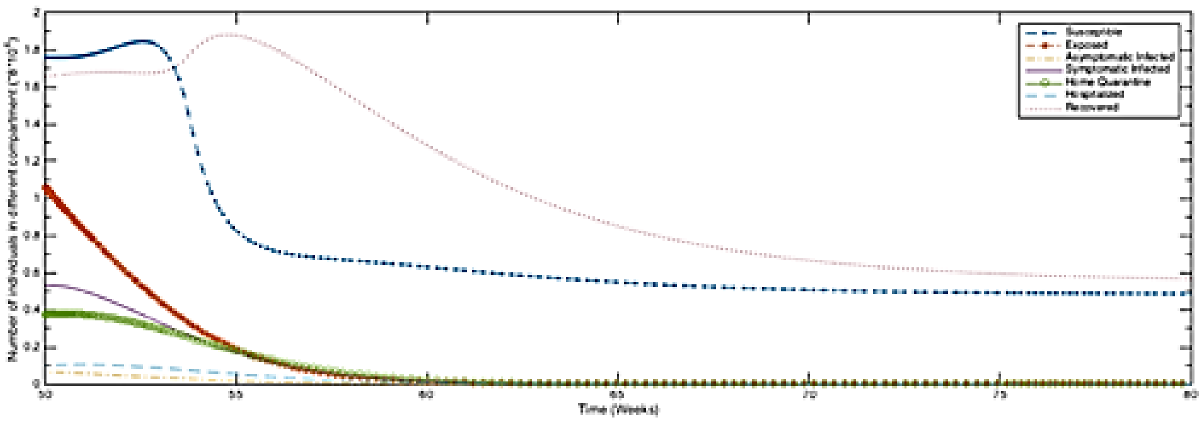

Fig. 4. Dynamics of the COVID-19 epidemic disease after vaccination as a control tool.

The results show that the parallel vaccination of individuals helps policymakers decrease the social and business costs of the lockdown policy. We determine the total doses of vaccine to control the epidemic disease based on different priority groups.

The total required number of vaccine doses for immunization of this group is 1,977,856 doses. For the other priority groups, we would have the following results:

Table 2. Results of epidemic control by vaccination as a control tool.

Priority Group	Required Number of Vaccine (Doses)		Time to Control (Days)
	90% Efficacy	78% Efficacy	
G1	1,487,605	1,808,012	About 49
G2	889,160	944,741	About 38
G3	787,500	819,000	About 31
Total	3,164,274	3,571,754	About 49

Therefore, the total number of vaccine doses will help the government control the epidemic without implementing a lockdown policy for retailers.

6 | Conclusion

This study examines the economic and social impacts of COVID-19 and highlights the importance of a well-planned vaccine allocation strategy during pandemics. By integrating an SEIR epidemic model and optimal control theory, the paper proposes a multi-objective framework that allocates vaccines among different priority groups, including shopkeepers and age-based groups, to minimize social costs, economic losses due to lockdowns, and vaccine procurement expenses. The model also accounts for varying vaccine efficacy and supplier pricing. Numerical results show that including non-age-based groups like retailers in prioritization decisions significantly improves outcomes. The study concludes that a parallel prioritization strategy enhances policymaking during pandemics and suggests future extensions involving other epidemic models, control measures (e.g., masking, distancing), and supplier selection criteria beyond cost.

Acknowledgments

The authors sincerely thank everyone who contributed to this research.

Funding

This research received no external funding.

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

References

[1] Gashaw, T., Hagos, B., & Sisay, M. (2021). Expected impacts of COVID-19: Considering resource-limited countries and vulnerable population. *Frontiers in public health*, *9*, 614789. https://doi.org/10.3389/fpubh.2021.614789

[2] Li, Z., Chen, Q., Feng, L., Rodewald, L., Xia, Y., & Yu, H. (2020). Active case finding with case management: The key to tackling the COVID-19 pandemic. *The lancet*, *396*(10243), 63–70. https://www.thelancet.com/journals/lancet/article/PIIS0140-6736(20)31278-2/fulltext

[3] Straetemans, M., Buchholz, U., Reiter, S., Haas, W., & Krause, G. (2007). Prioritization strategies for pandemic influenza vaccine in 27 countries of the European Union and the global health security action group: A review. *BMC public health*, *7*(1), 236. https://doi.org/10.1186/1471-2458-7-236%0A%0A

[4] Khodabandeh, P., Kayvanfar, V., Rafiee, M., & Werner, F. (2021). A bi-objective home health care routing and scheduling model with considering nurse downgrading costs. *International journal of environmental research and public health*, *18*(3), 900. https://doi.org/10.3390/ijerph18030900

[5] Tognotti, E. (2013). Lessons from the history of quarantine, from plague to influenza A. *Emerging infectious diseases*, *19*(2), 254. https://doi.org/10.3201/eid1902.120312

[6] Shamsi G., N., Ali Torabi, S., & Shakouri G., H. (2018). An option contract for vaccine procurement using the SIR epidemic model. *European journal of operational research*, *267*(3), 1122–1140. https://doi.org/10.1016/j.ejor.2017.12.013

[7] Imane, E., Jamal, B., & Abdelouahed, N. (2013). Dissemination of epidemic for SIR model. *Advances in mathematical sciences*, *7*(136), 6793–6800. http://dx.doi.org/10.12988/ams.2013.310594

[8] Abbasimehr, H., Paki, R., & Bahrini, A. (2022). A novel approach based on combining deep learning models with statistical methods for COVID-19 time series forecasting. *Neural computing and applications*, *34*(4), 3135–3149. https://doi.org/10.1007/s00521-021-06548-9%0A%0A

[9] Buckner, J. H., Chowell, G., & Springborn, M. R. (2020). Optimal dynamic prioritization of scarce COVID-19 vaccines. *Medrxiv*, 2009–2020. https://doi.org/10.1101/2020.09.22.20199174

[10] Choi, Y., Kim, J. S., Kim, J. E., Choi, H., & Lee, C. H. (2021). Vaccination prioritization strategies for COVID-19 in Korea: A mathematical modeling approach. *International journal of environmental research and public health*, *18*(8), 4240. https://doi.org/10.3390/ijerph18084240

[11] Korolev, I. (2021). Identification and estimation of the SEIRD epidemic model for COVID-19. *Journal of econometrics*, *220*(1), 63–85. https://doi.org/10.1016/j.jeconom.2020.07.038

[12] Guerstein, S., Romeo Aznar, V., Dekel, M., Miron, O., Davidovitch, N., Puzis, R., & Pilosof, S. (2021). The interplay between vaccination and social distancing strategies affects COVID19 population-level outcomes. *PLoS computational biology*, *17*(8), e1009319. https://doi.org/10.1371/journal.pcbi.1009319

[13] Shamsi Gamchi, N., Torabi, S. A., & Jolai, F. (2021). A novel vehicle routing problem for vaccine distribution using SIR epidemic model. *Or spectrum*, *43*(1), 155–188. https://doi.org/10.1007/s00291-020-00609-6%0A%0A

[14] Foy, B. H., Wahl, B., Mehta, K., Shet, A., Menon, G. I., & Britto, C. (2021). Comparing COVID-19 vaccine allocation strategies in India: A mathematical modelling study. *International journal of infectious diseases*, *103*, 431–438. https://doi.org/10.1016/j.ijid.2020.12.075

[15] Chapman, L. A. C., Shukla, P., Rodriguez Barraquer, I., Shete, P. B., León, T. M., Bibbins Domingo, K., & Lo, N. C. (2021). Comparison of COVID-19 vaccine prioritization strategies in the United States. *MedRxiv*, 2003–2021. https://doi.org/10.1101/2021.03.04.21251264%0A

[16] Kohli, M., Maschio, M., Becker, D., & Weinstein, M. C. (2021). The potential public health and economic value of a hypothetical COVID-19 vaccine in the United States: Use of cost-effectiveness modeling to inform vaccination prioritization. *Vaccine*, *39*(7), 1157–1164. https://doi.org/10.1016/j.vaccine.2020.12.078

[17] Laarabi, H., Rachik, M., El Kahlaoui, O., & Labriji, E. (2013). Optimal vaccination strategies of an SIR epidemic model with a saturated treatment. *Universal journal of applied mathematics*, *1*(3), 185–191. https://doi.org/10.13189/ujam.2013.010305 http://www.hrpub.org%0A