Research Annals of Industrial and Systems Engineering

www.raise.reapress.com

Res. Ann. Ind. Syst. Eng. Vol. 2, No. 4 (2025) 272–289.

Paper Type: Original Article

An Optimized Grey–Markov Computational Framework for Accurate Energy Demand Forecasting and Industrial Decision Support

Ghazale Azimi[1], Vahid Baradaran[1,*]

[1] Department of Industrial Engineering, NT.C., Islamic Azad University, Tehran, Iran; gh.azimi@iau-tnb.ac.ir; v_baradaran@iau.ac.ir.

Citation:

Received: 14 April 2025 *Revised: 21 June 2025* *Accepted: 16 September 2025*	Azimi, Gh., & Baradaran, V. (2025). An optimized grey–markov computational framework for accurate energy demand forecasting and industrial decision support. *Research annals of industrial and systems engineering, 2*(4), 272-289.

Abstract

Accurate long-term energy demand forecasting is essential for supporting strategic planning, infrastructure development, and resource allocation under increasing uncertainty. However, conventional forecasting methods often struggle to achieve reliable predictions when historical observations are limited, and energy consumption exhibits nonlinear and stochastic behavior. This study proposes a Simulated Annealing Markov Chain Grey Model (SA-MCGM) that integrates Grey System Theory, Markov Chain modelling, and Simulated Annealing (SA) optimization into a unified computational forecasting framework. The proposed approach combines deterministic trend modelling with probabilistic residual correction while simultaneously optimizing the grey model parameters and the number of Markov states to improve forecasting accuracy. The model is validated using annual energy consumption data for Asia from 2000 to 2022 and is compared with GM(1,1), the conventional Markov Chain Grey Model (MCGM), and ARIMA. Experimental results show that the proposed model achieves the best predictive performance during the testing period, reducing the Mean Squared Error (MSE) by 98.17% compared with the conventional MCGM and producing stable long-term forecasts through 2035. The findings demonstrate that simultaneous optimization of deterministic and stochastic model components significantly enhances forecasting robustness and generalization. The proposed SA-MCGM provides a computationally efficient forecasting framework that can support industrial engineering applications, including energy planning, infrastructure development, production planning, and strategic decision-making under uncertain conditions.

Keywords: Grey system theory, Simulated annealing, Markov chain, Energy demand forecasting, Computational modelling.

1 | Introduction

Accurate energy demand forecasting underpins nearly every major decision in energy system planning, from policy design to infrastructure investment and long-term resource allocation. The task is especially demanding in Asia, where rapid industrialization, population growth, and shifting consumption patterns produce

Corresponding Author: v_baradaran@iau.ac.ir
https://doi.org/10.22105/raise.v2i4.95

structural volatility that conventional planning tools struggle to anticipate. In 2021, global energy consumption reached 169,709 TeraWatt-hours (TWh), with Asia accounting for more than 60% of total demand [1]. Forecasting these trends with precision is essential not only for energy security but also for avoiding costly over- or under-investment in future supply capacity.

Energy forecasting also sits at the center of Sustainable Development Goal 7 (SDG7), which calls for universal access to affordable, reliable, sustainable, and modern energy [2]. Asia's position as the world's largest energy producer and consumer makes this goal particularly consequential: The region must reconcile continued reliance on conventional systems, such as coal and industrial grids, with an accelerating shift toward renewables, a balancing act that demands forecasting tools tailored to its specific volatility and data conditions [3]. A wide range of forecasting approaches has been developed to meet this need, spanning classical statistical methods, artificial intelligence, and hybrid techniques. Traditional methods such as ARIMA and other econometric models perform well under stable conditions but are ill-suited to capturing nonlinear trends and abrupt structural shifts. Hybrid grey-neural network structures have also been explored in adjacent forecasting domains, such as traffic flow prediction [4]. Modern data-driven techniques, including machine learning and deep learning approaches, offer considerably stronger predictive power [5], but they typically demand large, high-quality datasets and substantial computational resources, conditions that are frequently unmet in fast-changing or data-scarce regional contexts. This tension between model sophistication and data availability constitutes the central practical constraint motivating the present study. A related but distinct stream of work addresses industrial energy performance through integrated KPI frameworks rather than forecasting; Khezeli et al. [6], for instance, proposed a resilience-based performance evaluation model combining reliability, efficiency, and specific energy consumption to diagnose operational resilience in industrial systems, underscoring the broader relevance of energy-related indicators to industrial decision-making even outside a forecasting context. Uncertainty in energy-related decision-making has also been addressed through fuzzy set-based multi-criteria methods; for instance, Ejegwa et al. [7] developed a Q-rung Orthopair fuzzy distance measure to support the selection of reliable energy sources under imprecise criteria, illustrating a parallel line of work in which uncertainty modeling underpins energy decisions, distinct in technique but related in purpose to the grey-system approach adopted here. Grey system theory, introduced by Deng [8], offers a way around the data-availability constraint identified above. Grey forecasting models are designed explicitly for systems with sparse, incomplete, or imprecise data, making them well suited to regions where high-resolution historical records are limited. The standard GM(1,1) model, the most widely applied variant, has been used extensively for short-term energy forecasting because of its simplicity and its ability to extract underlying trends from small samples [9], with subsequent applications spanning electricity consumption, oil demand, CO_2 emissions, and gas production [10], [11]. Researchers have also worked to strengthen GM(1,1) itself, refining its background-value calculation [12] and introducing residual correction techniques [13] to improve the accuracy of the underlying data transformation.

Even with these refinements, GM(1,1) remains limited in its ability to represent the stochastic, non-stationary behavior typical of real energy markets. This has motivated the integration of Markov chains with grey models, yielding the Markov Chain Grey Model (MCGM), which models the residual sequence through state-transition probabilities and thereby captures uncertainty and temporal variability more effectively [14]. Zhou et al. [15] extended this line of work with a dynamic accumulation grey seasonal model to forecast renewable energy generation with improved accuracy under seasonal variability. Despite these gains, MCGM's performance remains highly sensitive to parameter selection, particularly the background value and the transition probability matrix, both of which are typically set empirically and can therefore fall short of optimal. Several optimization strategies have addressed individual components of this pipeline: Background-value optimization combined with Fourier-series residual correction [12], and further refinements to grey residual error correction [13], have each targeted one stage of the model, typically in isolation from its other parameters. Metaheuristic optimization has offered a further avenue for improvement. Particle Swarm Optimization (PSO) and related swarm-based metaheuristics have been used to tune grey-model parameters and initial conditions, and metaheuristic-optimized grey-Markov hybrids have also been explored outside the energy domain, such as Wu and Gao's use of a Cuckoo search algorithm to optimize a Grey-Markov model for tourism-demand forecasting [16]. More recent work has extended this metaheuristic-driven optimization to multivariate and hybrid grey structures: Ren et al. [17] developed a PSO-optimized multivariate grey model, GM(1,N), that incorporates compound extreme weather events as exogenous predictors of hydropower generation, achieving substantially lower forecasting error than conventional grey and LSTM benchmarks, while Wang et al. [18] proposed a fractional-derivative grey model with a self-adaptive reverse accumulation operator and PSO-tuned parameters, improving prediction accuracy on nonlinear, non-smooth energy consumption data in China. In a related direction, Li et al. [19] introduced a dynamic seasonal-buffered nonlinear grey model, optimized via the Marine Predator Algorithm, to jointly forecast multivariate renewable energy production and consumption under pronounced seasonality and small-sample conditions. Together, these studies confirm that metaheuristic optimization of grey model parameters, whether through PSO or other swarm-based algorithms, consistently improves forecasting accuracy across diverse energy contexts, yet none has applied Simulated Annealing (SA) to this task. Elsewhere in the broader forecasting literature, hybrid ARIMA-neural network models [20], regression-based methods for electric load forecasting[21], and artificial neural network ensembles for electricity demand [22] have each demonstrated strong predictive power but continue to face the same computational and data-volume constraints that motivate grey-system approaches in the first place.

Two gaps emerge from this body of work. First, despite Asia's outsized share of global energy demand, the geographical distribution of existing grey and hybrid forecasting studies skews heavily toward China and, to a lesser extent, other individual countries, leaving region-wide, cross-country Asian forecasting comparatively underexplored. Second, while parameter optimization for MCGM and related grey models has drawn growing attention, the reviewed techniques converge overwhelmingly on PSO and other swarm-based metaheuristics, including in the most recent multivariate and fractional-order extensions; the use of SA [23], a probabilistic global optimization method well suited to escaping local optima, has not been explored for tuning MCGM parameters. Existing, grey-based forecasting models offer a practical alternative to data-intensive machine learning methods in data-limited settings, but their accuracy remains constrained by empirically fixed parameters, and the specific combination of a data-frugal grey-Markov framework with a robust global optimization method has not yet been applied to long-range, region-wide energy forecasting for Asia. This study addresses that gap by proposing a hybrid Simulated Annealing Markov Chain Grey Model (SA-MCGM), which embeds SA into the MCGM framework to systematically optimize its parameters rather than relying on empirical or heuristic selection. Specifically, the study aims to: 1) develop an optimized GM capable of accurate prediction under data-limited conditions, 2) comparatively evaluate traditional and enhanced grey models against real-world Asian energy consumption data, and 3) generate reliable long-term energy forecasts to support strategic decision-making in a rapidly developing region. The GM(1,1), MCGM, and SA-MCGM models are applied to annual energy consumption data for Asia from 2000 to 2022, with performance assessed across a training period (2000-2015), a testing period (2016-2022), and long-term projections extending to 2035.

The remainder of this paper is organized as follows. Section 2 presents the methodology, detailing the theoretical formulation of the GM(1,1), MCGM, and SA-MCGM models and the practical procedure used to develop and optimize them. Section 3 reports the results of applying these models to Asian energy consumption data and discusses their significance for regional forecasting and planning. Section 4 concludes with a summary of the principal findings, the study's limitations, and directions for future research.

2 | Research Methodology

This study develops and validates an integrated forecasting framework, the SA-MCGM, which combines three components: The Grey Model GM(1,1), Markov Chain theory, and SA optimization. *Fig. 1* summarizes the overall workflow.

The foundational component is GM(1,1), which uses an accumulated generating operation to transform discrete, irregular time-series data into a monotonically increasing sequence suitable for modeling through a first-order differential equation. Markov Chain theory is then applied to the residuals of the GM(1,1) forecasts, using state transition probabilities to capture the stochastic fluctuations that a purely deterministic grey model cannot represent, yielding the MCGM. Finally, SA is applied to jointly optimize the parameters governing both components, rather than optimizing them independently, producing the proposed SA-MCGM framework.

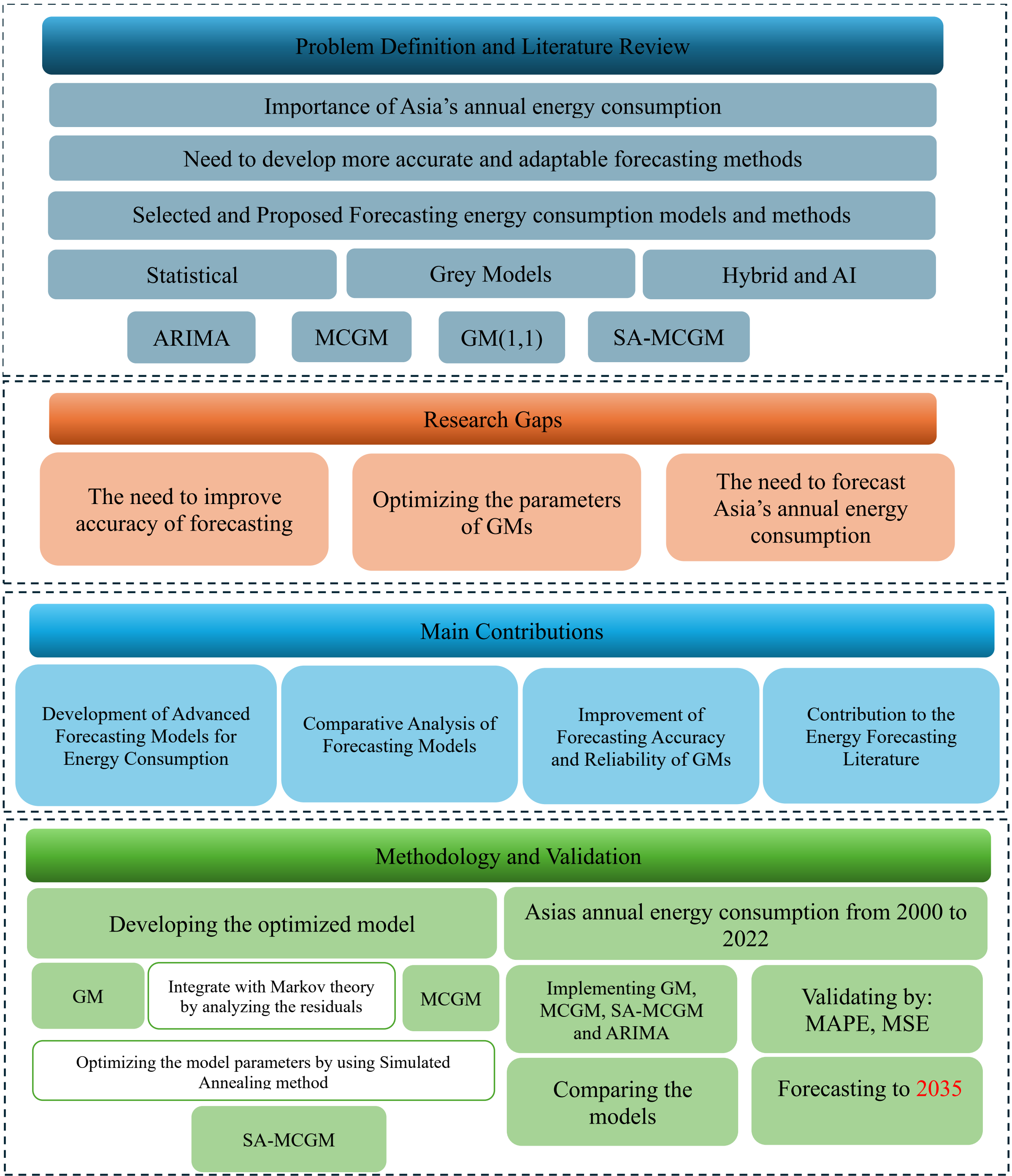


Fig. 1. Research methodology and contributions.

The empirical validation uses World Energy Agency data on energy consumption in the Asia region from 2000 to 2022, disaggregated across four vectors: natural gas, oil, electricity, and aggregate energy consumption. Four models are compared: The baseline GM(1,1), the MCGM, the proposed SA-MCGM, and ARIMA as an established benchmark. Forecasting accuracy is evaluated using Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE), computed both in aggregate and separately for each energy vector, with particular attention to predictive accuracy at the most recent observed point (2022) as an indicator of the models' reliability for future-facing forecasts.

Proposed models

This section develops the SA-MCGM in mathematical form, proceeding from the base grey model through the Markov-chain extension to the simulated-annealing optimization layer, with the underlying equations and reasoning presented at each stage.

2.1 | Grey Model

The (GM(1,1)) has three main stages: Summation operator, Inverse addition operator and forecasting.

Consider $x^{(0)}$ or x the initial time series (the real data) with n samples in *Vector (1)*.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)), \tag{1}$$

where, $x^{(0)}(i)$ is the time series value at *i*th time point. Using the Accumulated Generating Operator (AGO), the regular series is converted to the following time series in a monotonically increasing manner.

$$\mathbf{x}^{(1)} = (\sum_{i=1}^{1} x^{(0)}(i), \sum_{i=1}^{2} x^{(0)}(i), \dots, \sum_{i=1}^{n} x^{(0)}(i)). \tag{2}$$

At this stage, $x^{(0)}(i)$ can be calculated using the inverse operator with *Eqs. (3)* and *(4)*.

$$x^{(0)}(1) = x^{(1)}(1). \tag{3}$$

$$x^{(0)}(i) = x^{(1)}(i) - x^{(1)}(i-1) \quad \forall i = 2, \dots, n. \tag{4}$$

The grey model, GM(1,1), is formed using the First-degree differential equation as below:

$$\frac{d(x^{(1)})}{dt} + ax^{(1)} = b. \tag{5}$$

$$x^{(0)}(i) + az^{(1)}(i) = b \quad \forall i = 1, 2, \dots, n. \tag{6}$$

$$z^{(1)}(i) = \tfrac{1}{2}(x^{(1)}(i) + x^{(1)}(i-1)) \quad \forall i = 2, \dots, n. \tag{7}$$

This method does not have any mathematical proof and is called the whitening process at which i is a time point, a is improvement coefficient and b is the controller coefficient. The data of the GM(1,1) are shown in matrix form in *Relationships (8)-(10)*.

$$\mathbf{A} = \begin{pmatrix} -z^{(1)}(2) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}. \tag{8}$$

$$\mathbf{Y}_n = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}. \tag{9}$$

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix}. \tag{10}$$

Using the least squares method, the parameters of the model (a and b) are calculated as *Eq. (11)* [24].

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix} = (A'A)^{-1}A'Y_n'. \tag{11}$$

After calculating a and b, grey forecasting equation such as *Eq. (12)* [25] is obtained.

$$\hat{x}^{(0)}(i+1) = (x^{(0)}(1) - \frac{b}{a})(1-e^{a})e^{-ai} \quad \forall\, i = 1, 2, \ldots \tag{12}$$

So $x^{(0)}$ series is smoothened and $\hat{x}^{(0)} = \{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)\}$ will be the forecast series which calculated by *Eq. (12)*.

2.2 | Markov Chain Grey Model

While GM(1,1) captures general trends effectively, its accuracy deteriorates when the underlying data exhibit sharp, non-linear fluctuations, since the model has no mechanism for representing stochastic deviation from the deterministic trend it fits. The MCGM addresses this by treating the residual error between GM(1,1) predictions and observed values as a Markov process, allowing the model to adapt to the non-stationary behavior characteristic of energy consumption data.

The residual at each time point, quantifying the deviation between the GM(1,1) forecast and the observed value, is calculated as in *Eq. (13)*.

$$e(i) = x^{(0)}(i) - \hat{x}^{(0)}(i) \quad \forall\, i = 1, 2, \ldots. \tag{13}$$

The next stage is incorporating Markov chains for residual adjustment. The Markov chain component analyzes and adjusts the residuals, enhancing the accuracy of predictions by incorporating probabilistic transitions. Residuals $e(i)$ are partitioned into r states represented as S_j where j=1, 2, …, r. For each state, lower (L_j) and upper (U_j) bounderies are defined representing distinct ranges of error magnitudes. These boundaries are determined based on the distribution of $e(i)$, ensuring the states capture meaningful variations by *Eqs. (15)* and *(16)*.

$$S_j \in [L_j, U_j] \quad \forall\, j = 1, 2, \ldots, r. \tag{14}$$

$$L_j = \min_i e(i) + \frac{j-1}{r}(\max_i e(i) - \min_i e(i)) \;\forall\, j = 1, 2, \ldots, r. \tag{15}$$

$$U_j = \min_i e(i) + \frac{j}{r}(\max_i e(i) - \min_i e(i)) \;\forall\; j = 1, 2, \ldots, r. \tag{16}$$

The probability of transitioning from one state S_i to another S_j is represented in the transition matrix (P). The elements of transition matrix ($p_{ij}^{(m)}$) are calculated by *Eq. (17)* where m represents the probability of transitioning from state i to state j after m steps. In this context, $M_{ij}^{(m)}$ denotes the number of data points associated with the transition from state i to state j after m steps. M_i is the data which belongs to the ith state.

$$p_{ij}^{(m)} = \frac{M_{ij}^{(m)}}{M_i} \quad \forall i, j = 1, 2, \ldots, r. \tag{17}$$

Consequently, the matrix of transition probabilities among states is given by *Eq. (18)*.

$$P^{(m)}=\begin{bmatrix} p_{11}^{(m)} & p_{12}^{(m)} & \cdots & p_{1r}^{(m)} \\ p_{21}^{(m)} & p_{22}^{(m)} & \cdots & p_{2r}^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1}^{(m)} & p_{r2}^{(m)} & \cdots & p_{rr}^{(m)} \end{bmatrix}. \tag{18}$$

The Markov adjusted residual is added to the values obtained from *Eq.* (*12)* (GM(1,1) forecasted values) in order to improve the accuracy of the forecasting results which are calculated through *Relation (19*):

$$\hat{x}_{MCGM}(i+1)=\hat{x}^{(0)}+e_M(i). \tag{19}$$

In which $e_M(i)$ is the Markov-adjusted residual for time i and is calculated as *Eq. (20)*.

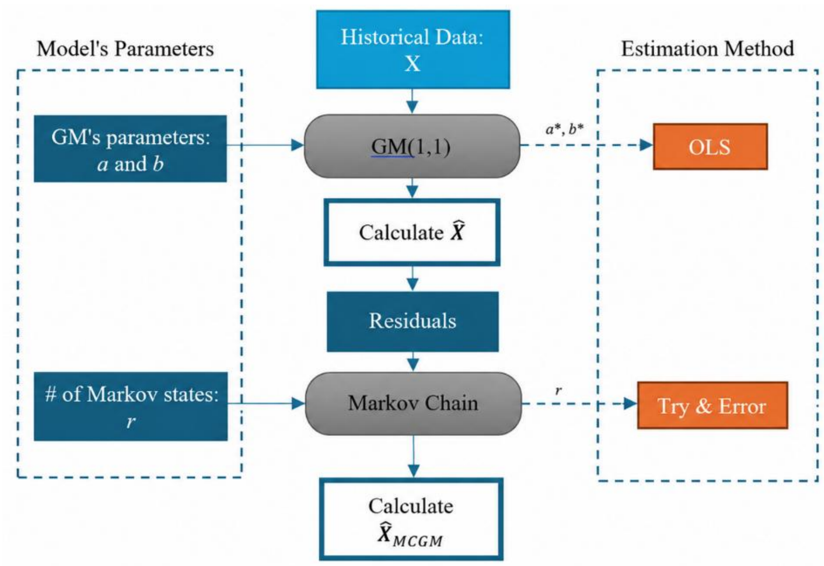
$$e_M(i)=\sum_{j=1}^{r}p_{ij}^{(m)}.\frac{\sum_{e(i)\in S_j}e(i)}{count(e(i)\in S_j)}. \tag{20}$$

In *Eq. (20)*, $count(e(i) \in S_j)$ is the count of $e(i)$ refers to the number of residual values $e(i)$ that belong to the state Sj . Specifically, the count is the number of times the residual values fall within the state S_j .

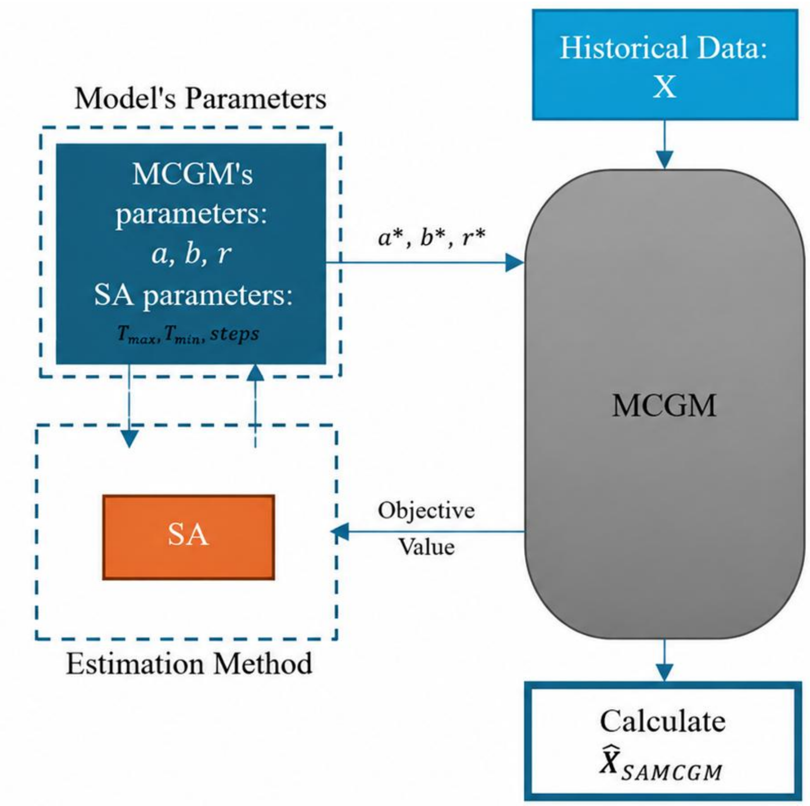
2.3 | Proposed SA-MCGM

2.3.1 | Motivation for optimization

The MCGM procedure described above is governed by three parameters: a and b, inherited from GM(1,1) and estimated via Ordinary Least Squares (OLS), and r, the number of Markov states, which is conventionally set by trial and error. *Fig. 2.a* depicts this standard MCGM pipeline. Despite its established performance, this configuration has three limitations that motivate the optimized framework developed here: 1) although a and b are globally optimal with respect to the GM(1,1) objective, that objective minimizes only the grey-model prediction error and does not account for how a and b interact with the subsequent Markov adjustment; their optimality is therefore only local with respect to the MCGM's actual forecasting objective, 2) the state parameter r is typically chosen through a subjective, analyst-dependent trial-and-error process, which offers no systematic way to evaluate its effect on model performance, and 3) because a, b, and r are estimated independently across two separate stages, the standard MCGM cannot represent trade-offs between them, even though all three jointly determine forecasting accuracy. The optimized MCGM developed in this study addresses these limitations by estimating a, b, and r jointly within a single framework, illustrated in *Fig. 2(b)*, in which small deviations from the OLS-optimal values of a and b are permitted where they improve the model's overall objective function once interaction with r is taken into account. Because the combined effect of a, b, and r on forecasting accuracy is nonlinear, gradient-based methods such as OLS are unsuitable for this joint optimization; the following section introduces SA as the optimization method used to resolve it.



a.



b.

Fig. 2. Schematic illustration of the forecasting methods; a. Conventional MCGM, and b. Proposed SA-MCGM with joint optimization of parameters.

2.3.2 | Simulated annealing optimization

SA is a probabilistic global optimization algorithm inspired by metallurgical annealing, in which a material is heated and then cooled slowly to remove defects and reach a stable low-energy structure [23]. By probabilistically accepting worse candidate solutions in early iterations and reducing that acceptance probability as the algorithm proceeds, SA can escape local optima and explore the solution space broadly, a property well suited to the nonlinear, interdependent parameter space of the MCG [26]. In this study, SA is used to jointly tune the development coefficient a, the grey action quantity b, and the number of Markov states r, treating all three as a single vector of decision variables rather than optimizing them in separate stages. The objective is to minimize forecasting error, measured as MAPE on the test dataset, as defined in *Eq. (21)*.

$$f(\theta) = MAPE = 100 \times \frac{1}{n'} \sum_{i}^{n'} \left| \frac{x^{(0)}(i) - \hat{x}_{SAMCGM}(i)}{x^{(0)}(i)} \right|. \tag{21}$$

Where $\theta = \{a, b, r\}$.

The SA algorithm proceeds iteratively, generating new candidate solutions by randomly perturbing one of the parameters at each iteration k, the change in the objective function is calculated as *Eq. (22)*.

$$\Delta f = f(x_{new}) - f(x_k). \tag{22}$$

If $\Delta f < 0$, the new candidate is accepted outright. Otherwise, the new candidate is accepted with the probability (P) given by the Metropolis criterion in *Eq.(23)* where T_k is the temperature at iteration k.

$$P = \exp(-\frac{\Delta f}{T_k}). \tag{23}$$

The temperature T_k decreases over the iterations according to a geometric cooling schedule. Our implementation updates the temperature as *Eq. (24)* where T_{max} is the initial temperature, T_{min} is the final temperature, steps is the total number of iterations:

$$T_k = T_{max} \cdot \left(\frac{T_{min}}{T_{max}} \right)^{\frac{k}{steps}}. \tag{24}$$

The algorithm terminates when either the temperature falls T_{min} or when the maximum number of iterations is reached. Unlike some approaches that incorporate a grid search for hyper parameter tuning, our method selects T_{max}, T_{min}, and steps based on preliminary experiments. This streamlined process directly integrates SA into the MCGM framework to iteratively refine the parameter estimates.

2.3.3 | Integration into simulated annealing markov chain grey model

The parameters returned by the SA procedure are embedded into the MCGM structure described in Section 4.2, yielding the proposed SA-MCGM. Because a, b, and r are optimized jointly rather than sequentially, the resulting parameter set accounts for the interaction between the grey-model trend component and the Markov-chain error correction, improving on both the local optimization and the trial-and-error state selection inherent in the standard MCGM. *Fig. 3* illustrates the resulting architecture: The initially high temperature permits broad exploration of the (a, b, r) parameter space, while the subsequent cooling schedule progressively concentrates the search on the regions yielding the lowest forecasting error, before the optimized parameters are fixed and used to generate the final SA-MCGM forecasts evaluated in the results and discussion section.

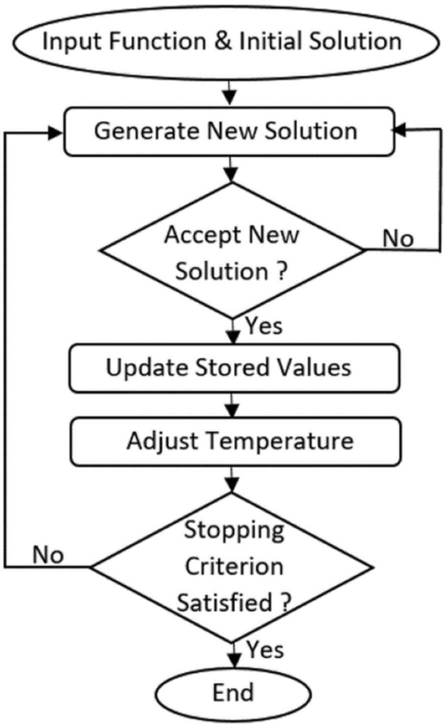


Fig. 3. Annealing simulation methodology.

3 | Discussion

3.1 | Data Description

Fig. 4 presents total energy consumption in Asia from 2000 to 2022, as reported by the World Energy Agency. Consumption rose from 59,584.29 TWh in 2000 to 121,234.56 TWh in 2022, nearly doubling over the 22-year period, with a minor dip in 2020 coinciding with the COVID-19-related economic slowdown. The corresponding energy mix, reported by the International Energy Agency, reflects the region's gradual diversification across fossil fuels, nuclear power, and renewables such as wind, solar, and hydropower.

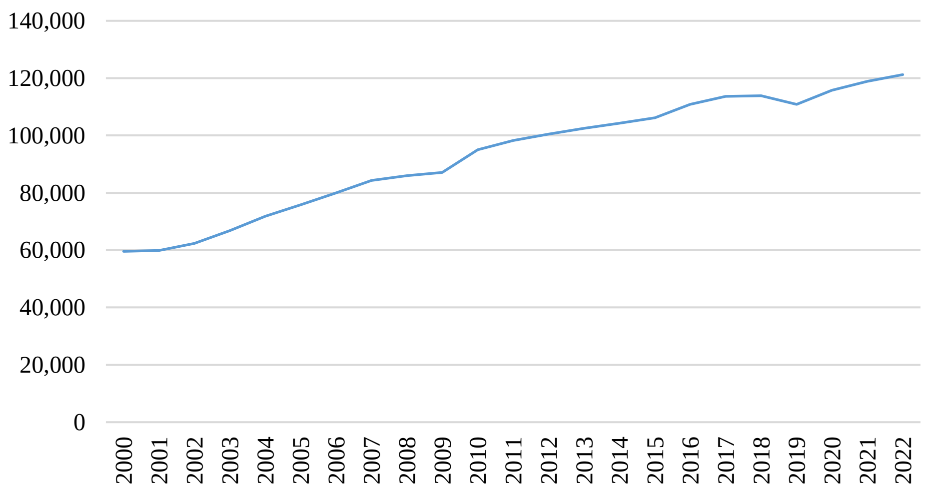


Fig. 4. Total energy consumption in asia TWh.

3.2 | SA Parameter Sensitivity

A sensitivity analysis was conducted on the three SA hyperparameters, the initial temperature T_{min} ,the final temperature T_{max} and the number of iterations (steps), evaluating MSE on a held-out test set across a predefined grid of values. Results are shown in *Fig. 5*. The sensitivity analysis was performed over a set of predefined values: $T_{max} \in \{25000, 50000, 100000\}, T_{min} \in \{2.5\times10^{-5}, 1\times10^{-5}, 1\times10^{-6}\}$, steps $\in \{2000, 5000, 10000\}$.

For each combination, the proposed optimizer model was executed to adjust the parameters of the SA. The MSE Performance metric was computed and shown in *Fig. 5* on a held-out test set to assess forecasting accuracy.

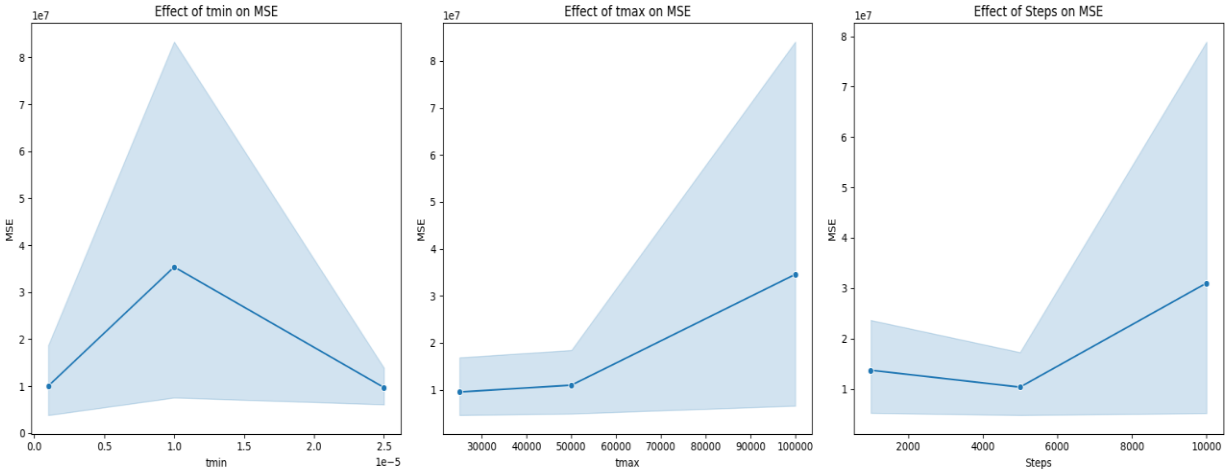


Fig. 5. The main effects of SA parameters on MSE.

The experimental analysis in *Fig. 5* reveals that optimal SA performance hinges on strategic parameter interplay. Higher T_{max} values (e.g., 100,000) promote exploration by escaping local optima early, but require pairing with sufficiently low T_{min} (e.g., 1×10^{-6}) to enable precise exploitation in later stages. Moderate iteration counts (Steps ≈ 5,000) balance computational efficiency and convergence quality, as excessive iterations yield diminishing returns, while insufficient steps hinder refinement. For instance, configurations like T_{max} = 25,000, T_{min} = 1×10^{-6}, and Steps = 10,000 achieved peak performance, whereas mismatched parameters (e.g., high T_{max} with inadequate T_{min}) led to instability. The findings advocate a calibrated hierarchy: Prioritize exploration with elevated T_{max}, enforce precision through minimized T_{min}, and align Steps with problem complexity to optimize both solution quality and computational practicality.

3.3 | Optimized Parameters

The implementation of the SA-MCGM incorporated carefully calibrated parameters to optimize its forecasting performance. After conducting the optimization using SA, the model achieved optimized parameter values that resulted in significantly improved forecasting accuracy. Under the optimal configuration, the development coefficient a was found to be approximately -0.0146, the grey action quantity b was approximately 83889.3901, and the number of Markov states r was set to 4. These values were determined through iterative refinement, which ensured that the model parameters were precisely tuned to minimize forecasting error.

Table1 compares the results of the SA-MCGM with the traditional MCGM, highlighting the differences in their respective parameter values and the objective function MSE. The development coefficient *a* in the SA-MCGM is slightly improved over the MCGM value of −0.0397. The grey action quantity b in SA-MCGM is 83,889.39, a 39.37% increase compared to MCGM's 60,194.88. A key difference between the models is the number of Markov states r. The SA-MCGM optimized *r* to 4 states, compared to 8 in MCGM, reflecting a decrease of 50%. This adjustment is expected to contribute to improved model performance by allowing for a more granular error correction process. The most significant difference lies in the MSE, where the SA-MCGM achieved an MSE of 3818842, a remarkable reduction of approximately 98.17% compared to the MCGM's MSE of 208622806.8. This substantial improvement in MSE confirms the effectiveness of the SA optimization process in refining the model's forecasting capabilities. These results demonstrate that the SA-MCGM, with its ability to more effectively explore the parameter space, offers a significant performance advantage over the traditional MCGM, especially in terms of reducing forecasting error.

Table 1. The parameters estimations of the simulated annealing markov chain grey model vs MCGM.

Parameter/Objective Fun.	MCGM	SA-MCGM	Percentage of Changes
Development coefficient (a)	-0.0397	-0.0146	63.45%
Grey Action quantity (b)	60,194.88	83,889.39	39.37%
Markov states (r)	8	4	-50%
MSE	208,622,807	3,818,842	-98.17%

3.4 | Forecasting Performance

To assess the performance of the proposed model, *Table 3* compares the actual energy consumption values with those predicted by different methods across both training and testing datasets. The statistical method ARIMA was employed to forecast the actual energy consumption values. Following the analysis of autocorrelation and partial autocorrelation function plots, an ARIMA model of order one (AR(1)) was implemented on the data. The column GM(1,1) shows the predicted annual energy consumption values using the first-order grey model with the parameters listed in *Table 1*. After applying the MCGM method to the actual values, predictions were calculated using this method. For the MCGM approach, a trial-and-error method was utilized to determine the optimal value of r that minimizes the MAPE, ensuring the highest prediction accuracy. This iterative process highlights the importance of parameter tuning in achieving reliable and precise forecasting results. Utilizing the parameters from the SA-MCGM method in *Table 1*, the annual energy consumption values in Asia were computed using the proposed method.

Table 2. Forecasted data of annual energy consumption for the training and test periods.

Type	Year	Actual	ARIMA	GM(1,1)	MCGM	SA-MCGM
	2000	59,584.29	59,665.53	59584.29	59584.29	59584.29
	2001	59,889.37	59,624.89	63815.11	63815.11	85380.72
	2002	62,392.24	59,930.18	66396.89	66396.89	86636.11
	2003	66,854.23	62,434.76	69083.13	69083.13	87909.95
	2004	71,825.82	66,899.79	71878.04	71363.98	66897.61
	2005	75,891.77	71,874.77	74786.02	74192.44	68209.19
	2006	80,013.89	75,943.49	77811.66	78327.74	75912.89
Training data	2007	84,326.69	80,068.42	80959.7	80311.29	70890.5
	2008	85,974.54	84,384.16	84235.1	83507.31	85006.46
	2009	87,116.03	86,033.13	87643.02	85804.62	86396.9
	2010	95,044.98	87,175.40	91188.81	88285.2	87807.79
	2011	98,244.03	95,109.75	94878.06	91932.29	76493.75
	2012	100,495.80	98,310.98	98716.56	96791.74	77946.43
	2013	102,524.33	100,564.28	102710.36	100701.84	98752.87
	2014	104,239.56	102,594.20	106865.73	105483.83	100733.76
	2015	106,183.37	104,310.60	111189.22	107245.04	102443.54
	2016	110,821.04	106,255.73	115687.63	112355.75	110812.94
	2017	113,648.12	108,057.89	120368.03	117036.15	112375.6
	2018	113,896.58	108,131.53	125237.78	121905.91	113961.24
Test data	2019	110,828.52	108,205.21	130304.55	126972.68	115570.2
	2020	115,738.95	108,278.95	135576.31	132244.44	117202.81
	2021	118,851.23	108,352.74	141061.35	137729.48	118859.43
	2022	121,234.56	108,426.58	146768.3	143436.43	120540.4

Following the precise calculation of the Average Percentage Error (APE) for each method from 2000 to 2022 in two different periods, the percentage of absolute deviation is illustrated in *Fig 6*. The analysis highlights SA-MCGM as the most robust model for long-term forecasting. While it initially shows high errors (e.g., 42.56% in 2002), its adaptive mechanisms drive significant improvement over time, achieving near-perfect accuracy (0.007% in 2022) during testing. In contrast, ARIMA exhibits moderate but inconsistent performance (0.14–10.56%), while GM(1,1) and MCGM degrade sharply in later years (up to 21.06% and 18.31%, respectively). SA-MCGM's ability to minimize errors over extended horizons, coupled with its superior test-phase stability, positions it as the optimal choice for scenarios requiring long-term precision, despite initial calibration challenges.

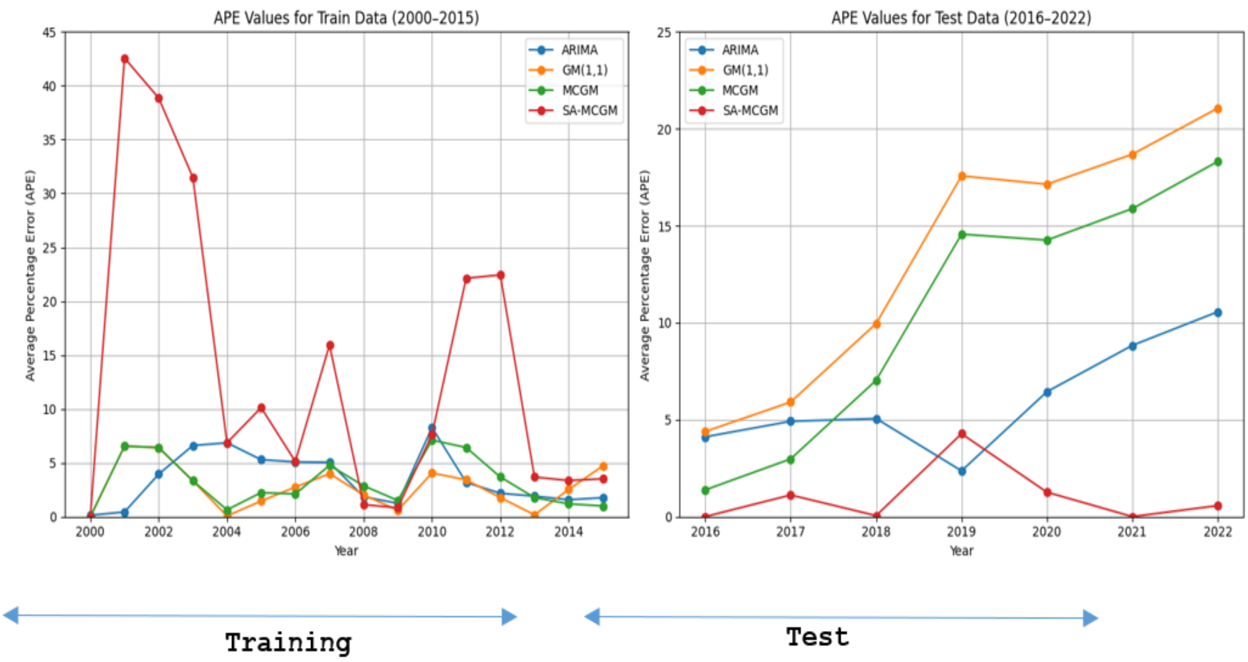
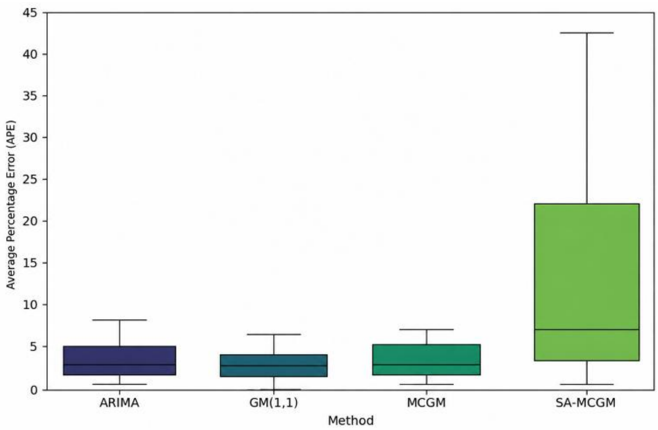
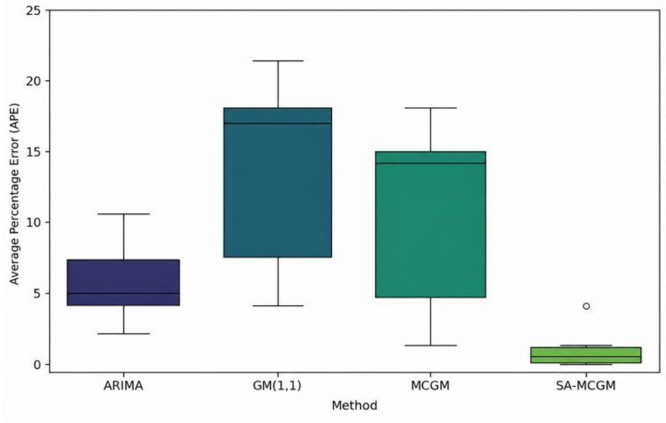


Fig. 6. The absolute percentage error for each forecasting method.

To better compare the methods, two Analysis of Variance (ANOVA) test were performed. The results of the ANOVA on the APEs indicate that the performance of all four methods differs at the significant level of 0.000 in two datasets (learning and testing data). The results of ANOVA as box plots of APE to compare the algorithms are shown in *Fig. 7*.



a.



b.

Fig. 7. The Interval Plot of ANOVA test for comparisons of forecasting methods; a. APE for learning data, b. APE for testing data.

The comparative performance of the prediction methods is further illustrated in the side-by-side boxplots of the APE for the training and testing datasets. As evident from the plots, the proposed SA-MCGM algorithm demonstrates superior performance for both datasets compared to the other methods, ARIMA, GM(1,1), and MCGM. In the training data, SA-MCGM shows higher variability and an extreme outlier (APE of 42.56), but in the testing data, it exhibits consistently low APE values (ranging from 0.007 to 4.28, with a median of approximately 0.57), indicating remarkable stability and accuracy. The other performance metrics, including MAPE, Mean Absolute Error (MAE), and MSE, are presented in the table below for the two periods of training and testing data. The analysis results highlight a significant relative improvement of the SA-MCGM model compared to the ARIMA, GM(1,1), and MCGM prediction methods, particularly in the testing phase, where it outperforms the others with lower and more consistent errors.

Table 3. Model performance metrics.

Metric	Model	Training	Testing
MAPE	GM	2.74%	13.53%
	MCGM	3.22%	10.63%
	SA-MCGM	13.48%	1.04%
MSE	GM	7,405,457	302,223,168
	MCGM	10,721,300	208,622,844
	SA-MCGM	189,841,068	3,818,842
MAE	GM	71	15,712
	MCGM	71	-1,111
	SA-MCGM	-1,475	615

The performance metrics presented in *Table 3* offer a comprehensive comparison of the forecasting models GM(1,1), MCGM, and SA-MCGM, highlighting their efficacy across training and testing phases. GM(1,1) demonstrates a low training MAPE of 2.74%, yet its testing MAPE escalates to 13.53%, with MSE surging from 7,405,457 to 302,223,168, indicative of overfitting and poor generalization to unseen data. Similarly, MCGM, while maintaining a modest training MAPE of 3.22%, sees a testing MAPE of 10.63% and an MSE increase to 208,622,844, suggesting a moderate overfitting tendency that compromises its predictive reliability. In stark contrast, SA-MCGM exhibits a training MAPE of 13.48% and an elevated MSE of 189,841,068, which might initially suggest underfitting; however, its testing performance is exceptional, with MAPE plummeting to 1.04% and MSE reducing to 3,818,842. This remarkable improvement underscores SA-MCGM's superior generalization capability, a critical attribute in forecasting applications. The success of SA-MCGM can be attributed to its hybrid methodology, which integrates SA to optimize the grey forecasting framework. This approach appears to mitigate the overfitting pitfalls observed in GM(1,1) and MCGM by prioritizing robustness over excessive fit to training data, as evidenced by its tightly clustered testing errors. Furthermore, the MAE metrics reinforce this conclusion: While GM(1,1) and MCGM show substantial deviations in testing (15,712 and -1,111, respectively), SA-MCGM's testing MAE of 615 reflects minimal absolute error, affirming its precision. These results align with contemporary trends in forecasting literature, where hybrid models leveraging optimization techniques consistently outperform traditional approaches by balancing complexity and adaptability.

Although GM(1,1) and MCGM offer competitive training performance, their inability to maintain accuracy in testing renders them less reliable for practical applications. SA-MCGM, by contrast, excels where it matters most, delivering dependable forecasts on new data. This robustness, coupled with its methodological sophistication, positions SA-MCGM as the preeminent choice among the evaluated models, offering a compelling case for its adoption in scenarios demanding high predictive accuracy and stability.

3.5 | Long-Term Forecasting (2023–2035)

The projected values for annual energy consumption in Asia from 2023 to 2035 have been forecasted using GM(1,1), MCGM, and SA-MCGM methods, utilizing the optimal parameter values calculated in previous sections. These projections are illustrated in *Fig. 8*. It is anticipated that the energy consumption will reach approximately 144780.04 units by the year 2035 using the proposed method.

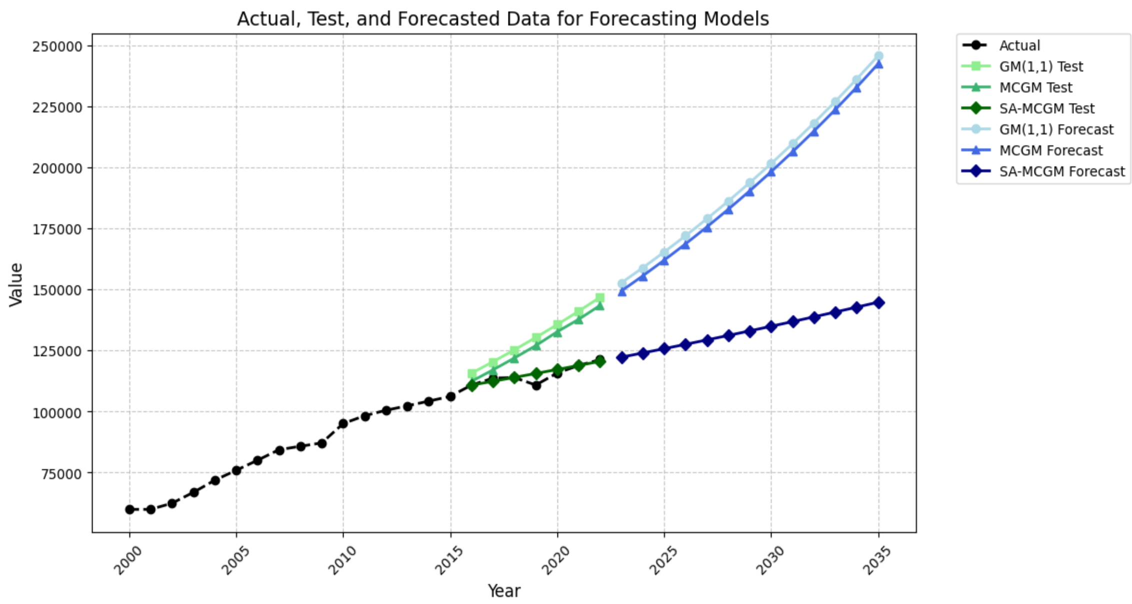


Fig. 8. The results data for test and forecasted of the models.

The analysis of the forecasting models GM(1,1), MCGM, and SA-MCGM, spanning the period from 2023 to 2035, reveals statistically significant disparities in their predictive outcomes. A one-way ANOVA yielded an F-statistic of 71.19 with a p-value effectively approaching zero ($p < 0.001$), indicating robust evidence to reject the null hypothesis of equality among the group means. Examination of the temporal trends demonstrates that GM(1,1) exhibits the most pronounced growth, projecting a value exceeding 245,000 by 2035, while MCGM follows a comparable trajectory, albeit with a slightly tempered increase, culminating at approximately

242,000. In contrast, SA-MCGM displays a markedly conservative profile, stabilizing at around 144,000 with minimal incremental change over the study period. This divergence is further corroborated by the box plot analysis (*Fig. 9*), which illustrates greater variability and higher median values for GM(1,1) and MCGM, indicative of their expansive forecasting ranges, whereas SA-MCGM presents a narrower interquartile range and lower median, suggesting a more constrained predictive scope. These findings underscore significant methodological differences with potential implications for applied forecasting contexts, highlighting the necessity for careful model selection in research and policy applications where projected outcomes may substantially influence strategic planning and resource allocation.

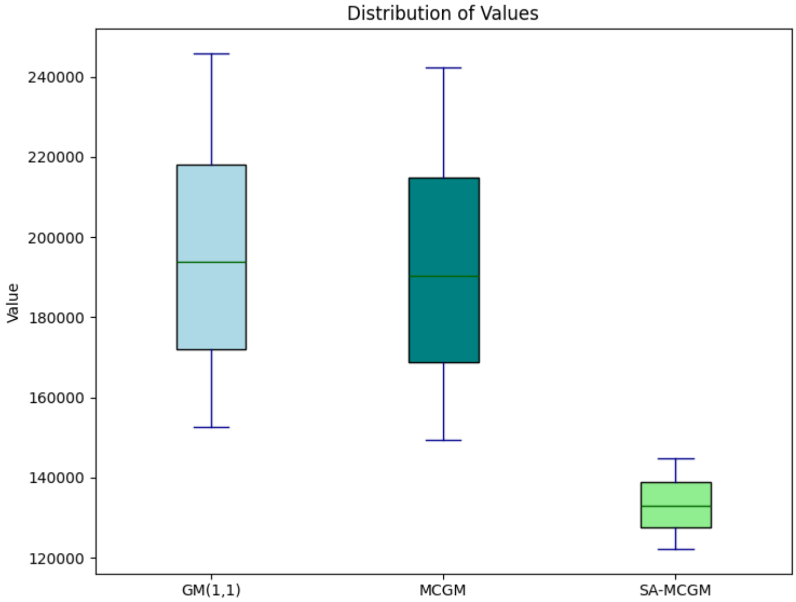


Fig. 9. The box plot of ANOVA test for comparisons of forecasted data.

This divergence has a direct practical implication: Because GM(1,1) and MCGM extrapolate the historical upward trend largely unchecked, they imply substantially higher future infrastructure and supply requirements than SA-MCGM, which instead reflects the flattening pattern already visible in the most recent testing-period data (2016–2022). Given SA-MCGM's demonstrated testing-period accuracy in Section 3.4, its more conservative long-range projection is the more credible basis for capacity and policy planning, though the magnitude of the gap between models underscores the sensitivity of long-horizon forecasts to model choice.

4 | Conclusion

This study developed SA-MCGM, a hybrid forecasting model that jointly optimizes the parameters of the MCGM using SA and applied it to annual energy consumption in Asia from 2000 to 2022. Relative to the standard MCGM, joint optimization reduced the development coefficient a from −0.0397 to −0.0146, increased the grey action quantity b from 60,194.89 to 83,889.39, and reduced the number of Markov states r from 8 to 4, together yielding a 98.17% reduction in MSE (from 208,622,806.80 to 3,818,842).

In out-of-sample testing, SA-MCGM achieved a MAPE of 1.04% and an MSE of 3,818,842, outperforming ARIMA, GM(1,1), and MCGM, all of which showed signs of overfitting to the training trend. Long-term projections to 2035 diverge substantially across models: GM(1,1) and MCGM extrapolate continued steep growth (exceeding 240,000 units), while SA-MCGM projects a comparatively stable trajectory (approximately 144,780 units), consistent with the flattening pattern observed in the most recent testing years.

These findings indicate that jointly optimizing grey-model and Markov-chain parameters, rather than estimating them sequentially, yields a meaningful improvement in generalization for long-range energy forecasting. The study is limited to aggregate consumption data for a single region (Asia) over a fixed historical window, and the model relies solely on historical consumption data without exogenous variables. Future work could extend SA-MCGM to other regions and energy vectors, incorporate exogenous predictors such as GDP, urbanization, or electrification trends, and evaluate its performance against additional metaheuristic optimizers beyond SA.

Acknowledgments

Acknowledgments enable you to thank all those who have helped in carrying out the research. Careful thought needs to be given concerning those whose help should be acknowledged and in what order. The general advice is to express your appreciation concisely and to avoid strong emotive language.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

Funding

This section indicates any support that is not included in the Author Contribution or Funding sections. This may include administrative and technical support or in-kind donations (e.g., materials used for experiments).

Data Availability

We encourage all authors to make their research data available. Please indicate where the data supporting the reported findings can be found, including links to publicly archived datasets that were analyzed or generated during the study. If no new data were generated or if data are not available due to privacy or ethical restrictions, an explanation is still required. The suggested Data Availability Statements were collected in the DAS file.

Conflicts of Interest

The authors declare no conflict of interest. Authors must declare any personal circumstances or interests that could be considered to have an inappropriate influence on the presentation or interpretation of the reported research findings. Any role of funders in the design of the study, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results must be disclosed in this section. If this is not the case, please indicate, "Funders played no role in the design of the study, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results." You can see more details in the CID file.

References

[1] BP's Website. (2022). *Annual report and form 20-F 2022*. https://www.bp.com/api/files/6cqieuqhq4no/master/10C5j6Wi2CkktKj1WH3OuG/7635c09e4bf89081483a3ff319c70ed9/bp-annual-report-and-form-20f-2022.pdf

[2] Nations, U. (2021). *Policy Brief 9: SDG 7 in Asia and the Pacific*. https://sdgs.un.org/sites/default/files/2021-05/POLICY BRIEF 9 - SDG 7 IN ASIA AND THE PACIFIC.pdf

[3] International Energy Agency. (2021). *Energy Transition in Southeast Asia.* https://www.iea.org/reports/energy-transition-in-southeast-asia/

[4] Chen, S. Y., Qu, G. F., Wang, X. H., & Zhang, H. Z. (2003). Traffic flow forecasting based on grey neural network model. *Proceedings of the 2003 international conference on machine learning and cybernetics* (Vol. 2, pp. 1275-1278). IEEE. https://doi.org/10.1109/ICMLC.2003.1259684

[5] Sharma, G., Dhore, M. L., Jansirani, D., Bala Jeshurun, S., Sathi, G., Sherje, N., & Vivek, V. (2024). AI and machine learning applications in predicting energy market prices and trends. *E3S web of conferences* (Vol. 591, p. 01002). EDP Sciences. https://doi.org/10.1051/e3sconf/202459101002

[6] Khezeli, M., Seidi, M., & Najafi, S. E. (2025). Performance evaluation using the hybrid resilience threshold based on reliability, efficiency and specific energy consumption. *Journal of applied research on industrial engineering*, *12*(2), 334–361. https://doi.org/10.22105/jarie.2025.464447.1624

[7] Ejegwa, P. A., Anum, M. T., Kausar, N., Nwokoro, C. O., & Cagin, T. A. (2025). A novel Q-rung Orthopair fuzzy distance metric for energy source selection. *Journal of applied research on industrial engineering*, *12*(3), 429–464. https://doi.org/10.22105/jarie.2025.530199.1821

[8] Ju-Long, D. (1982). Control problems of grey systems. *Systems & control letters*, *1*(5), 288–294. https://doi.org/10.1016/S0167-6911(82)80025-X

[9] Kayacan, E., Ulutas, B., & Kaynak, O. (2010). Grey system theory-based models in time series prediction. *Expert systems with applications*, *37*(2), 1784–1789. https://doi.org/10.1016/j.eswa.2009.07.064

[10] Huang, L., Liao, Q., Zhang, H., Jiang, M., Yan, J., & Liang, Y. (2021). Forecasting power consumption with an activation function combined grey model: A case study of China. *International journal of electrical power & energy systems*, *130*, 106977. https://doi.org/10.1016/j.ijepes.2021.106977

[11] Ma, X., Qi, C., Ye, L., Yang, D., & Hong, J. (2017). Life cycle assessment of tungsten carbide powder production: A case study in China. *Journal of cleaner production*, *149*, 936–944. https://doi.org/10.1016/j.jclepro.2017.06.167

[12] Liu, D., Li, G., Chanda, E. K., Hu, N., & Ma, Z. (2019). An improved GM (1.1) model with background value optimization and Fourier-series residual error correction and its application in cost forecasting of coal mine. *Gospodarka surowcami mineralnymi*, *35*(3), 75–98. https://doi: 10.24425/gsm.2019.128528

[13] Xiao, L., Chen, X., & Wang, H. (2021). Calculation and realization of new method grey residual error correction model. *Plos one*, *16*(7), e0254154. https://doi.org/10.1371/journal.pone.0254154

[14] Kazemi, A., Modarres, M., Mehregan, M., & others. (2013). Energy demand forecast of Iran's industrial sector using Markov chain Grey model. *The international journal of humanities*, *20*(1), 1–12. https://dor.isc.ac/dor/20.1001.1.25382640.2013.20.1.6.3

[15] Zhou, W., Jiang, H., & Chang, J. (2023). Forecasting renewable energy generation based on a novel dynamic accumulation grey seasonal model. *Sustainability*, *15*(16), 12188. https://doi.org/10.3390/su151612188

[16] Sun, X., Sun, W., Wang, J., Zhang, Y., & Gao, Y. (2016). Using a Grey--Markov model optimized by Cuckoo search algorithm to forecast the annual foreign tourist arrivals to China. *Tourism management*, *52*, 369–379. https://doi.org/10.1016/j.tourman.2015.07.005

[17] Ren, Y., Xia, L., & Wang, Y. (2023). Forecasting China's hydropower generation using a novel seasonal optimized multivariate grey model. *Technological forecasting and social change*, *194*, 122677. https://doi.org/10.1016/j.techfore.2023.122677

[18] Wang, Y., Chi, P., Nie, R., Ma, X., Wu, W., & Guo, B. (2022). Self-adaptive discrete grey model based on a novel fractional order reverse accumulation sequence and its application in forecasting clean energy power generation in China. *Energy*, *253*, 124093. https://doi.org/10.1016/j.energy.2022.124093

[19] Li, X., Li, J., Zhao, Y., & Zhou, S. (2025). A novel discrete multivariable grey model with seasonal time-lag effect for clean energy generation forecasting. *Energy*, 137678. https://doi.org/10.1016/j.energy.2025.137678

[20] Zhang, G. P. (2003). Time series forecasting using a hybrid ARIMA and neural network model. *Neurocomputing*, *50*, 159–175. https://doi.org/10.1016/S0925-2312(01)00702-0

[21] Lee, G.-C. (2024). A regression-based method for monthly electric load forecasting in South Korea. *Energies*, *17*(23), 5860. https://doi.org/10.3390/en17235860

[22] Azadeh, A., Ghaderi, S. F., & Sohrabkhani, S. (2008). Annual electricity consumption forecasting by neural network in high energy consuming industrial sectors. *Energy conversion and management*, *49*(8), 2272–2278. https://doi.org/10.1016/j.enconman.2008.01.035

[23] Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, *220*(4598), 671–680. https://doi.org/10.1126/science.220.4598.671

[24] Li, X., Dang, Y., & Zhao, J. (2010). An optimization method of estimating parameters in GM (1, 1) Model. In *Advances in grey systems research* (pp. 341–347). Springer. https://doi.org/10.1007/978-3-642-13938-3_29

[25] Saxena, A. (2021). Grey forecasting models based on internal optimization for Novel Corona virus (COVID-19). *Applied soft computing*, *111*, 107735. https://doi.org/10.1016/j.asoc.2021.107735

[26] Aarts, E., & Korst, J. (1989). *Simulated annealing and Boltzmann machines: A stochastic approach to combinatorial optimization and neural computing*. John Wiley & Sons, Inc.. https://dl.acm.org/doi/abs/10.5555/61990