Research Annals of Industrial and Systems Engineering

www.raise.reapress.com

Research Expansion Alliance

Res. Ann. Ind. Syst. Eng. Vol. 3, No. 3 (2026) 160–169.

Paper Type: Original Article

A SBM DEA Model of Performance Generation via Input–Output Ratios under Pseudo-Returns to Scale

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Citation:

Received: 06 March 2026 *Revised: 13 May 2026* *Accepted: 20 July 2026*	Dehghan-Chenari, S., & Hossienzadeh-Lotfi, F. (2026). A SBM DEA model of performance generation via input–output ratios under pseudo-returns to scale. *Research Annals of Industrial and Systems Engineering, 3*(3), 160-169.

Abstract

Data Envelopment Analysis with Ratio measures (DEA-R) provides an effective framework for evaluating the performance of decision-making units based on ratio indicators. However, existing DEA-R studies generally treat outputs as a homogeneous set and overlook systems in which outputs play different functional roles and exhibit distinct scaling behaviors. This study introduces a novel ratio-based technology with two categories of outputs. The first category is combined with inputs to construct ratio-based performance indicators, whereas the second category represents outcome outputs generated through the utilization of these performance structures. A new axiomatic framework is developed, and the concept of pseudo-returns to scale is extended to the ratio–output space. The proposed approach characterizes pseudo-returns to scale within ratio-based technologies. The framework provides a new perspective for analyzing organizations in which performance indicators and outcome indicators respond differently to resource utilization and operational development.

Keywords: Data envelopment analysis, Ratio-based technology, Pseudo-returns to scale, Ratio performance indicators, Outcome outputs.

1 | Introduction

Data Envelopment Analysis (DEA) is one of the most widely used nonparametric methodologies for evaluating the relative efficiency and performance of Decision-Making Units (DMUs). In many practical applications, performance assessment is based not only on absolute input and output values but also on ratio-based indicators. In such situations, DEA with ratio measures (DEA-R) provides an appropriate framework for measuring and comparing the performance of organizational units. This approach has been widely applied in various fields, including higher education, healthcare systems, banking, energy management, and organizational performance evaluation.

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https://doi.org/10.22105/raise.vi.98

Existing DEA-R studies have primarily focused on efficiency measurement, network models, slack-based models, inverse DEA formulations, and technologies involving undesirable outputs. Despite these developments, most existing approaches treat outputs as a homogeneous set and do not consider the distinct functional roles and scaling behaviors of different output categories. In many real-world systems, some outputs directly contribute to the formation of performance indicators, whereas other outputs represent outcomes generated through the utilization of these performance structures.

For example, in higher education institutions, the ratio of research funding to faculty members may serve as a performance indicator, while publications, citations, and patents represent outcomes generated by the underlying performance structure. Similarly, in healthcare systems, certain indicators reflect the efficiency of resource utilization, whereas service quality and patient satisfaction represent outcome-oriented achievements. In such environments, performance indicators and outcome outputs do not necessarily follow the same scaling pattern, and outcome outputs may respond to operational changes at rates that differ from those of performance indicators.

Despite the importance of this issue, the concepts of returns to scale and, in particular, pseudo-returns to scale have not been investigated within ratio-based technologies. Furthermore, no set of axioms has been developed for analyzing scaling behavior in systems whose outputs consist of distinct categories of performance outputs and outcome outputs. This research gap highlights the need for a new theoretical framework for analyzing scaling behavior in ratio-based technologies.

This study seeks to address the following research questions:

I. How can a ratio-based technology be constructed for systems involving two distinct categories of outputs, namely performance outputs and outcome outputs?

II. What set of axioms is required to characterize the production technology and feasibility conditions in ratio-based environments with heterogeneous output categories?

III. How can the concept of pseudo-returns to scale be defined and characterized within a ratio-based technology where outcome outputs respond differently to changes in ratio-based performance indicators?

The main contributions of this study are as follows:

I. Introducing a novel ratio-based production technology with two distinct categories of outputs, including performance outputs and outcome outputs.

II. Developing a new set of axioms to characterize feasibility and production behavior in ratio-based environments with heterogeneous outputs.

III. Extending the concept of pseudo-returns to scale from conventional input–output technologies to ratio-based technologies.

IV. Establishing a theoretical framework for analyzing the responsiveness of outcome outputs to changes in ratio-based performance indicators.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on DEA with ratio measures and pseudo-returns to scale. Section 3 introduces the proposed ratio-based technology with two categories of outputs and presents its underlying axioms. Section 4 develops the concept of pseudo-returns to scale within the proposed framework and investigates its theoretical properties. Section 5 provides a numerical example to illustrate the applicability of the proposed approach and to demonstrate its theoretical implications. Finally, Section 6 concludes the paper and suggests directions for future research.

2 | Literature Review

DEA was first introduced by Abraham Charnes et al. [1], who proposed a nonparametric framework for evaluating the relative efficiency of DMUs with multiple inputs and outputs. Subsequently, Rajiv D. Banker et al. [2] extended the original DEA framework by introducing the BCC model, enabling the assessment of technical efficiency and returns to scale under variable returns-to-scale assumptions. These studies established the theoretical foundations for efficiency measurement and scale analysis in DEA.

As performance evaluation increasingly relied on ratio indicators, researchers began investigating the relationship between ratio analysis and DEA. Thanassoulis et al. [3] compared DEA and ratio analysis in healthcare performance evaluation and demonstrated that ratio analysis alone is insufficient for comprehensive benchmarking and target setting. Later, Despić et al. [4] formally introduced DEA with Ratio Measures (DEA-R) by incorporating output–input ratios as outputs within the DEA framework, thereby establishing a direct theoretical connection between DEA and ratio analysis.

A second stream of research focused on methodological developments for handling ratio data within DEA models. Emrouznejad and Amin [5] showed that conventional DEA assumptions may generate biased efficiency scores when ratio variables are present and proposed modifications to the convexity assumption. Mozaffari and Garami [6] developed several input-oriented DEA-R formulations, including envelopment, additive, enhanced Russell, and central resource allocation models. Subsequently, Mozaffari et al. [6] proposed DEA-R models for estimating cost and revenue efficiency and demonstrated their applicability when only ratio information is available. In another study, Mozaffari et al. [6] investigated DEA-R models without explicit inputs and established connections with output-oriented DEA formulations. These studies considerably expanded the theoretical and practical scope of DEA-R.

A third research stream concentrated on improving the discriminatory power and robustness of ratio-based efficiency evaluation. Azizi [7] proposed a worst-efficiency analysis framework based on inefficient frontiers and ratio measures. Hatami Marbini and Tolou [8] highlighted limitations of conventional DEA in dealing with ratio measures and developed modified DEA models that preserve key DEA properties while accommodating ratio data.

More recent studies have extended DEA-R to complex production environments. Garami et al. [9] developed a two-stage multi-criteria DEA-R framework for network structures and addressed issues such as pseudo-inefficiency and weight discrimination. Garami et al. [10] integrated slack-based measures into DEA-R and proposed SBM-DEA-R models capable of computing efficiency and super-efficiency while overcoming efficiency underestimation problems.

Another active research direction concerns uncertainty, network structures, and inverse DEA-R formulations. Ostovan et al. [11] developed DEA and DEA-R models for two-stage network systems under fuzzy data environments. Sohrabi et al. [12] proposed inverse DEA-R models based on multiple-objective linear programming for estimating input and output levels in ratio-data settings. Gerami et al. [13] extended DEA-R to network supply-chain evaluation and introduced ratio-based network DEA models that account for internal structures and managerial preferences.

Several recent contributions have further expanded the applicability of DEA-R. Hosseinzadeh Lotfi et al. [14] developed two-stage network DEA-R models based on SBM and central resource allocation frameworks. In another study, Hosseinzadeh Lotfi et al. [14] proposed a hybrid DEA–ratio analysis approach to determine the contribution of individual ratios to overall efficiency. They also introduced DEA-R applications for sustainability assessment in supply chains and formulated DEA-R models without explicit inputs for evaluating commercial banks. Wanak et al. [15] proposed stochastic network DEA-R models for ratio data using black-box, free-link, and fixed-link structures. Sohrabi et al. [12] developed DEA-R-based resource-sharing and strategic alliance models using inverse DEA-R methodologies. More recently, Keshtkar et al. [16] proposed two-stage non-radial DEA and DEA-R models for systems involving both desirable and undesirable outputs.

1.2 | Research Gap

Although these studies have substantially enriched the DEA-R literature, several important research gaps still remain. For example, most existing studies have primarily focused on efficiency measurement, benchmarking, network structures, inverse models, uncertainty, and sustainability-related applications. Moreover, outputs are generally treated as a homogeneous set, whereas in many real-world systems outputs perform different functions and play distinct roles within the production process. In addition, the existing DEA-R literature has not distinguished between performance outputs used in the construction of ratio-based indicators and outcome outputs generated through the utilization of those performance structures. Furthermore, although the concepts of returns to scale and pseudo-returns to scale have been extensively investigated in conventional DEA environments, no corresponding framework has yet been developed for ratio-based technologies.

Therefore, a significant research gap exists regarding the development of ratio-based technologies with heterogeneous output categories and the characterization of pseudo-returns to scale within such environments. This study seeks to address this gap by proposing a novel ratio-based technology, establishing its underlying axioms, and extending the concept of pseudo-returns to scale to ratio–output production spaces.

3 | A Proposed Ratio-Based Technology with Two Categories of Outputs

In conventional production technologies, inputs are transformed directly into outputs, and the production process is represented within the input–output space. However, in many real-world organizations, performance is not assessed solely through absolute input and output levels. Instead, performance is often evaluated using ratio-based indicators that reflect the effectiveness of resource utilization. Furthermore, not all outputs play the same role within the production process. While some outputs directly participate in the construction of performance indicators, others represent outcomes generated through the utilization of those performance structures.

Motivated by this observation, we introduce a new ratio-based technology involving two categories of outputs. Consider a set of n DMUs, where each DMU utilizes m inputs to generate two distinct categories of outputs. Let $X=(x_1,...,x_m)$ denote the vector of inputs, $Y=(y_1,...,y_s)$ denote the first category of outputs, referred to as performance outputs, and $Z=(z_1,...,z_t)$ denote the second category of outputs, referred to as outcome outputs. In fact, the union of Y and Z constitutes the set of output indicators.

The first category of outputs is combined with the inputs to construct ratio-based performance indicators. These indicators describe the efficiency of resource utilization and represent the operational performance structure of the organization. The second category of outputs reflects the outcomes generated through the utilization of this performance structure and may exhibit a scaling behavior different from that of the performance indicators. Accordingly, for each DMU, a ratio-performance vector is defined as $R=(r_1,...,r_s)$, where each ratio indicator is constructed from the corresponding input-output relationship. In general, $(r_k=\frac{x_i}{y_k}; k=1,...,s \ \& \ i=1,...,m)$. The proposed production technology is therefore represented in the ratio–output space rather than the conventional input–output space. Specifically, the technology consists of feasible combinations of ratio-performance indicators and outcome outputs and is denoted by $T_R=\{(R,Z)\}$. This representation enables the investigation of production behavior and scale properties in environments where performance is evaluated through ratio indicators and organizational outcomes are generated as a consequence of those performance structures. In the following subsection, a set of axioms is introduced to characterize the proposed technology and establish its fundamental properties.

The proposed technology differs from conventional input–output technologies because performance is represented through ratio-based indicators and outcome outputs are generated through the utilization of these performance structures. Consequently, traditional returns-to-scale concepts may not adequately characterize the scale behavior of such systems. To address this issue, we adopt the Pseudo-Returns to Scale (P-RTS) axiom introduced by Dehghan Chenari et al. [2]. This axiom shows that, in ratio-based environments, scale behavior depends on the relative development of inputs and outputs rather than on the absolute magnitude of expansion or contraction. Based on this perspective, the fundamental properties of the proposed technology are established through a set of underlying axioms presented in the following subsection.

4 | Axiomatic Foundations and Mathematical Formulation of the Proposed Technology

4.1 | Axiomatic Foundations

The proposed technology is characterized by the following axioms, which define feasibility, production behavior, and scale properties in the ratio–output space.

Axiom 1. Observability: For every observed DMU, the corresponding ratio-performance indicators and outcome outputs are assumed to be feasible and therefore belong to the technology set.

Axiom 2. Ratio-Based Free Disposability: If a ratio–output combination is feasible, then any combination with weaker ratio-performance indicators and lower outcome outputs is also considered feasible.

Axiom 3. Outcome Monotonicity: For a given level of ratio-performance indicators, any level of outcome outputs lower than an observed feasible level is also attainable and belongs to the technology set.

Axiom 4. Convexity: Any convex combination of feasible activities is also feasible. Therefore, the technology set contains all weighted combinations of observed feasible activities.

Axiom 5. Minimization Principle: For a given level of outcome outputs, DMUs are assumed to seek the smallest attainable ratio-performance indicators. Consequently, the technology is oriented toward minimizing ratio measures while maintaining the desired outcome outputs.

Based on the five axioms presented above, the proposed ratio-based technology can be constructed as follows:

$$T_R = \left\{ \begin{pmatrix} r \\ z \end{pmatrix} \middle| r_j \geq \sum_{j=1}^{n} \lambda_j r_{kj}, k = 1,\ldots,s, z \leq \sum_{j=1}^{n} \lambda_j z_{pj}, p = 1,\ldots,t, \sum_{j=1}^{n} \lambda_j = 1, j = 1,\ldots,n \right\}. \tag{1}$$

4.2 | Pseudo-Returns to Scale in the Proposed Ratio-Based Technology

Having established the ratio-based technology, the next step is to investigate its scale behavior. Unlike conventional production technologies, where returns to scale are examined through proportional changes in inputs and outputs, the proposed technology operates in a ratio–output space. Consequently, scale behavior depends on the relative development of ratio-performance indicators and outcome outputs.

Let α denote the development factor associated with the ratio-performance structure and β denote the development factor associated with the outcome outputs. The scale behavior of the proposed technology depends on the relative magnitudes of these development factors.

Accordingly, for any feasible activity belonging to the technology set, the scaled activity can be represented by applying the corresponding development factors to the technology. Under the proposed framework, outcome outputs are assumed to respond more strongly to development than ratio-based indicators. Therefore, $\beta > \alpha \geq 1$. Proposed technology *Model (2)* is constructed by applying the axiom of pseudo-increasing returns to scale (P-IRS) to *Model (1)*.

$$T_{R-PIRS} = \left\{ \begin{pmatrix} r \\ z \end{pmatrix} \middle| r_j \geq \sum_{j=1}^{n} \alpha \lambda_j r_j, z \leq \sum_{j=1}^{n} \beta \lambda_j z_j, \sum_{j=1}^{n} \lambda_j = 1, j = 1,\ldots,n, \beta > \alpha \geq 1 \right\}. \tag{2}$$

Under the proposed technology, P-IRS arise when the development factor associated with outcome outputs exceeds that of ratio-based indicators. In this situation, improvements in ratio-based performance indicators lead to more than proportional improvements in outcome outputs. Therefore, the technology exhibits an amplification effect, whereby the benefits generated through the ratio-performance structure are transferred to outcome outputs at an increasing rate. This behavior indicates that the system can transform performance improvements into outcome achievements with increasing effectiveness.

For completeness, Pseudo Constant Returns to Scale (P-CRS) and Pseudo Decreasing Returns to Scale (P-DRS) may also be defined according to alternative relationships between the development factors. However, the remainder of this study focuses on the pseudo increasing returns to scale case, which constitutes the primary setting of the proposed technology.

4.3 | Development Model Based on Proposed Ratio-Based Technology with P-IRS

Based on the ratio-based production technology defined in *Model (2)*, the efficiency evaluation *Model (3)* is developed. The proposed model operates directly in the ratio space T_{R-PIRS} and is fully consistent with the underlying axioms, particularly feasibility and pseudo-returns to scale. Its objective is to determine the relative position of each DMU with respect to the efficient frontier defined in the ratio domain.

$$
\begin{aligned}
&\text{Min}\frac{1-\frac{1}{s}\sum_{k=1}^{s}\frac{s_k^-}{r_o}}{1+\frac{1}{p}\sum_{p=1}^{t}\frac{s_p^+}{z_o}},\\
&\sum_{j=1}^{n}\alpha\lambda_j r_{kj}+s_k^-=r_o, \qquad k=1,\dots,s, \qquad (c-1),\\
&\sum_{j=1}^{n}\beta\lambda_j z_{pj}-s_p^+=z_o, \qquad p=1,\dots,t, \qquad (c-2),\\
&\sum_{j=1}^{n}\lambda_j=1, \qquad (c-3),\\
&1\le\alpha<\beta,\lambda_j\ge 0, j=1,\dots,n, s_k^-\ge 0, k=1,\dots,s, s_p^+\ge 0, p=1,\dots,t.
\end{aligned} \tag{3}
$$

The objective function minimizes the weighted average of input slacks while simultaneously maximizing the weighted average of output slacks, reflecting the efficiency of a DMU in the ratio-based production framework. Constraint (c-1) represents the representation of ratio-based indicators with the development coefficient α on the efficient frontier within the framework of DEA. Constraint (c-2) pertains to the second type of outputs (i.e., outcome outputs) and is specified with the development coefficient β. Constraint (c-3) is the convexity constraint (convexity condition).

The proposed *Model (3)* is always feasible since a trivial solution can be constructed by setting a single intensity variable equal to one and all slack variables to zero. Under this construction, all constraints are satisfied, ensuring a non-empty feasible region.

Theorem 1. *Model (3)* is feasible, and the efficiency score lies within the interval [0,1].

Proof: Feasibility is guaranteed because, for any unit under evaluation, one can select the intensity weight $\lambda_j=1$ for the corresponding unit and set all slacks $s_k^-=s_p^+=0$. Under this choice:

$$\sum_{j=1}^{n}\alpha\lambda_j r_{kj}+s_k^-=r_o,\ \sum_{j=1}^{n}\beta\lambda_j z_{pj}-s_p^+=z_o \text{ and the convexity constraint } \sum_{j=1}^{n}\lambda_j=1 \text{ is satisfied.} \tag{4}$$

Regarding the efficiency score, the objective is defined as a ratio of the form:

$$\text{efficiency} = \frac{1 - \text{average input slacks}}{1 + \text{average output slacks}}. \tag{5}$$

Since all slacks are non-negative, the numerator is ≤ 1, and the denominator is ≥ 1. Consequently, the efficiency score always satisfies within the interval [0,1]. It equals 1 if and only if all slacks are zero. This explanation completes the proof.

5 | Example

To illustrate the application of the proposed model, it was implemented on 10 DMUs characterized by two inputs and three outputs. The outputs are divided into two categories: (y_1) and (y_2) belong to the first category, while (z) belongs to the second category. Moreover, the ratios of inputs to the first-category outputs generate the data associated with the first set of constraints. These ratios are adjusted by a scaling coefficient that is greater than or equal to one. In addition, the development factor associated with the second category of outputs is greater than the development factor of the ratio-based indicators, implying that the second-category outputs evolve at a higher rate than the ratio-based measures. The dataset is presented in *Table 1*.

Table 1. Data set.

DMUs	Inputs		Outputs of First Category		Ratio input to Output		Output of Second Category
	x_1	x_2	y_1	y_2	$r_1=x_1/y_1$	$r_2=x_2/y_2$	z
DMU1	10	8	15	12	0.666	0.666	18
DMU2	12	9	22	24	0.375	0.409	20
DMU3	9	6	14	10	0.642	0.6	22
DMU4	11	10	13	13	0.687	0.769	19
DMU5	13	11	25	27	0.65	0.407	24
DMU6	8	7	12	9	0.666	0.777	17
DMU7	14	10	22	18	0.636	0.555	28
DMU8	10	9	25	12	0.4	0.75	18
DMU9	12	8	17	15	0.705	0.533	26
DMU10	9	8	13	10	0.692	0.8	16

By solving the model in GAMS, the efficiency scores of all units were obtained, along with the corresponding input and output development factors, providing a detailed assessment of performance. In *Table 3*, the results of applying *Model 3* to the 10 DMUs are presented.

Table 2. Efficiency scores and development factors of 10 DMUs based on the model.

DMUs	Efficiency Z	Benchmarking		
		$r_1=x_1/y_1$	$r_2=x_2/y_2$	z
D1	0.23212	0.375	0.409	20
D2	1	0.375	0.409	20
D3	0.35245	0.375	0.409	20
D4	0.06926	0.375	0.409	20
D5	1	0.65	0.407	24
D6	0.0909	0.375	0.409	20
D7	0.41879	0.375	0.409	20
D8	0.52828	0.375	0.409	20
D9	0.34224	0.375	0.409	20
D10	0.02224	0.375	0.409	20

The model is linearized and solved using the Charnes-Cooper transformation [17]. In addition, an upper bound is defined for the development coefficient associated with the second category of outputs to ensure model tractability and stability. The proposed model is then implemented and solved under these conditions. The optimal development coefficient obtained for the generation of the outputs was 1.5, indicating that the outputs can expand at a rate 1.5 times that of the ratio-based performance indicators.

1.5 | Managerial implications

The results indicate that only D2 and D5 are located on the efficient frontier and serve as benchmark units. However, most inefficient DMUs are projected toward the performance structure of D2, as the benchmark values of the performance ratios ($r_1 = 0.375$; $r_2 = 0.409$) and the final output ($z = 20$) are repeatedly observed for the majority of inefficient units. This finding suggests that, within the proposed technology, efficiency depends less on the absolute levels of inputs and outputs and more on forming an appropriate ratio-based performance structure. In other words, the performance ratios constitute the foundation for generating the final outcomes, and improvements in these ratios can lead to enhanced overall organizational performance.

Furthermore, the substantial variation in efficiency scores reflects significant differences among DMUs in their ability to transform performance structures into final outcomes. Units such as D4, D6, and particularly D10, which exhibit very low efficiency scores, are considerably distant from the efficient frontier and require substantial improvements in resource utilization and performance structure management. The presence of both D2 and D5 on the efficient frontier also indicates that multiple pathways exist for achieving efficient performance. That is, organizations may attain full efficiency through different combinations of ratio-based performance indicators and outcome outputs. This finding highlights the flexibility of the proposed technology and its capability to identify alternative patterns of successful organizational performance.

6 | Conclusion

In this study, a novel ratio-based technology was developed within the SBM-DEA framework, in which outputs are classified into two categories: performance outputs and outcome outputs. In the proposed technology, performance outputs are combined with inputs to construct ratio-based indicators that represent the operational performance structure of an organization. Outcome outputs are subsequently generated through the utilization of these performance structures. Therefore, unlike conventional DEA models that analyze production processes in the input–output space, the proposed framework models production activities in a ratio–output space, providing a suitable mechanism for evaluating organizations whose performance is assessed through ratio-based indicators rather than absolute input and output levels.

To investigate the developmental behavior of the proposed technology, the concept of pseudo-returns to scale was extended by incorporating differentiated development coefficients for ratio-based performance indicators and outcome outputs. The results revealed that the estimated development coefficient associated with the outcome outputs was equal to 1.5 and exceeded the corresponding development coefficient of the ratio-based indicators. This finding suggests that improvements in ratio-based performance structures can lead to more-than-proportional growth in outcome outputs. In other words, final outcomes can expand at a rate greater than the growth rate of the underlying performance indicators, indicating the presence of a developmental advantage in the transformation of performance structures into organizational outcomes. Furthermore, the efficiency assessment demonstrated that efficient units not only possess superior ratio-based performance structures but also exhibit a greater ability to transform these structures into desirable outcomes. From a managerial perspective, these findings highlight that improving ratio-based performance indicators can serve as an effective strategy for accelerating the growth of outcome outputs and enhancing overall organizational performance.

Several avenues for future research may be considered. First, the proposed framework can be extended to accommodate undesirable outputs and environmental factors, thereby increasing its applicability to sustainability and environmental performance assessment problems. Second, the concept of pseudo-returns to scale may be generalized to dynamic and network DEA structures in which multiple layers of ratio-based indicators interact over time. Third, researchers can incorporate uncertainty and data variability through fuzzy, interval, or stochastic DEA approaches. Finally, future studies may investigate alternative forms of ratio construction and development mechanisms to further explore the relationship between performance structures and outcome generation in complex production systems.

Acknowledgments

The authors are grateful to all individuals who contributed to the completion of this study through their assistance and constructive input.

Funding

The authors declare that this study was conducted without financial support from any external organization.

Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.

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